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ABSTRACT

These instructional materials were produced as part of the project, Developing Computational Estimation Materials for the Middle Grades. The introduction to these eighth grade materials covers the following: why teach estimation; how the materials were developed; and how the lessons are organized. The 15 lessons that follow are designed to teach such estimation strategies as front-end estimation, compatible numbers, clustering, and rounding in lessons with whole numbers, fractions, mixed numbers, decimals, and percents. Each lesson plan includes objectives, teacher background, suggestions for teaching the lesson, acceptable answers for exercises, and six worksheets for student use. (MNS)

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Computational Estimation Instructional Materials

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Grade 8

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WHY TEACH COMPUTATIONAL ESTIMATION?

Estimation has long been recognized as a valuable, useful skill in many vocations and in daily life. With the growing use of calculators and computers it is vital that people be able to judge the reasonableness of an answer. Also there are many instances where an estimate is all that is required to make an important decision. Despite the importance of estimation it has seldom received serious attention in curriculum materials and teachers have had few resources available for supplementing their own ideas. Evidence of students' performance on estimation indicates that most students do not have high proficiency with it, nor do even good estimators have a strong level of confidence in their ability to estimate.

HOW WERE THESE MATERIALS DEVELOPED?

In recent years there has been a renewed interest in this topic, including an increase in research on students' thinking on estimation tasks and on learning specific estimation strategies. This curriculum development project has been built upon this growing body of research. These materials were developed as part of a National Science Foundation project to provide a teaching resource for middle grades and junior high school. This particular set of lessons is designed to provide systematic instruction of effective estimation strategies in Grade 8. Other sets are available for Grades 6 and 7.

These lessons have been successfully used in schools. (A report documenting their effectiveness is available from any of the authors.) This package of lessons has been field tested and reflects helpful suggestions that have been provided by many teachers and students.

The extensive field testing of these materials revealed that there are many ingredients necessary to helping students become proficient estimators. One of these ingredients which you will want to keep in mind as you use these materials is the development of a proper mental set for estimation. This includes:

1. Recognition that estimation is important and useful.

2. Awareness that many situations require only an estimate.

3. Recognition that there are many ways to obtain reasonable estimates.

HOW ARE THE MATERIALS ORGANIZED?

Fifteen lessons have been written for each grade. Each lesson has the following components:

1. Objectives The objective(s) for each lesson is stated at the top of the first page of teacher notes.

2. Teacher Background This section discusses the strategies taught in the lesson in detail. Sometimes it also provides some additional background notes to help teachers better understand the approaches used.

3. Teaching the Lesson Brief suggestions for teaching the lesson are provided. The major portion of each lesson is developed through overhead transparencies. You will need to make the transparencies from the paper copies provided in these materials. The transparencies often provide real-world settings requiring estimation. They also present key steps highlighting each strategy along with examples for students to try under your direction. We think you will find the transparencies very useful in your teaching. They highlight the main ideas and focus students' attention on the key steps.

4. Using the Exercises Brief comments and suggestions for using the assignment sheets are given.

5. Student Assignment Sheets A two-page worksheet is provided for each lesson. Each worksheet also provides some real-world applications of estimation. These should be started in class and completed as homework. As time permits, discussion of selected exercises the following day will promote estimation thinking and awareness of the many ways of obtaining a reasonable estimate.

WHAT ARE THE LESSONS?

The lesson titles for the 8th grade materials are given here. In most cases the titles are descriptive, however please refer to the specific lessons for a more comprehensive explanation of the topic.

GRADE 8 LESSON OUTLINE

Lesson 1.	Introduction to Estimation Front-End Strategy for Addition of Whole Numbers
Lesson 2.	Front-End Strategy for Addition of Decimals Compatible Number Strategy for Addition
Lesson 3.	Clustering Strategy
Lesson 4.	Front-End Strategy for Subtraction Compatible Number Strategy for Subtraction
Lesson 5.	Rounding Strategy for Multiplication
Lesson 6.	Front-End Strategy for Multiplication
Lesson 7.	Front-End Strategy for Division
Lesson 8.	Compatible Number Strategy for Division
Lesson 9.	Addition of Fractions and Decimals Near 0, ½, and 1
Lesson 10.	Estimating a Fraction of a Number
Lesson 11.	Estimating Multiple Operation Problems
Lesson 12.	Multiplication and Division of Numbers Near Powers of Ten
Lesson 13.	Addition of Mixed Fractions and Decimals
Lesson 14.	Estimating Percent of a Number (Near Powers of Ten)
Lesson 15.	Estimating Percent of a Number (Near Common Percents)

USING THE MATERIALS

The process of developing students' estimation competency is a long one. As they have repeated contacts with estimating and as they develop competence with specific techniques for obtaining an estimate, students will gain skill and confidence. Although your students may not reach a high level of competency in one year, progress will be made through systematic instruction.

You have an important role to play in developing students' ability to estimate. Initially many students may show resistence toward estimating. Other students will welcome the opportunity to share self-developed estimation strategies. Through discussion of thinking strategies with students and the encouragement of students' sharing their own thinking for a problem, you can help them gain new appreciation for the estimation process.

We think these lessons emphasize the important components of estimation skill and will be most interested in learning about your experience in using them. Everything needed for classroom use has been provided in this package. You are welcome to copy any or all of these materials to share with colleagues or students. Good luck to you!

NSF ESTIMATION
GRADE 8 - LESSON 1

OBJECTIVES : Create an awareness for the usefulness of estimation.

Establish the idea that there are many different yet successful ways to make estimates.

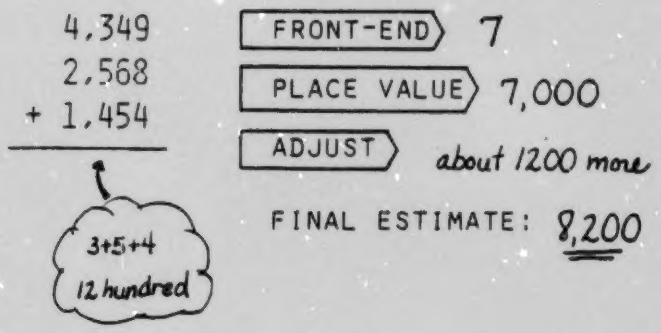
Introduce and develop the front-end addition strategy for whole numbers.

TEACHER BACKGROUND

This is an important lesson for students. It will help them begin to appreciate the importance of the skill of estimating. This appreciation for estimation is necessary as students begin to learn various estimation strategies but it takes time to develop. Relaying your own everyday experiences with estimation will help students develop this appreciation.

The estimation technique introduced in this lesson is the FRONT-END strategy for addition. The FRONT-END strategy is an alternative to rounding which is found in many textbooks. Rounding will be developed later in this series of lessons.

The FRONT-END process begins by focusing on the lead digits of a problem. These lead digits are added and the appropriate place value is attached. At this stage an UNDER-ESTIMATE is obtained. Students should be encouraged to ADJUST this initial estimate by focusing on the next group of digits. The example illustrates the entire process.



The emphasis in this process is on working with the most important digits-the lead digits (front-end digits). Students should recognize that these digits give the best information about the problem. In this lesson problems will be presented in both horizontal and vertical formats. An example of the FRONT-END process for horizontal format is given:

2,328 + 479 + 5,104 + 28

1. CHOOSE THE IMPORTANT DIGITS: 2 + 5 = 7

2. ATTACH THE APPROPRIATE PLACE VALUE: 7,000

3. ADJUST: 300 + 400 + 100
800 more

ESTIMATE: 7,800

The crucial step when working with such a problem is to determine which digits to use in Step 1. Encourage students to think about the digits which represent the largest place value. Also illustrated in this problem is the fact that one addend (28) was not used in the estimation process. Students will quickly see that it represents a very small amount relative to the whole problem.

In the last example the adjust step was accomplished by mentally adding the hundreds digits (3 + 4 + 1). Another acceptable method for adjusting might simply be recognizing that about another 1000 is needed, producing an estimate of 8000. Don't worry too much about the closeness of the adjust step. Accept any reasonable amount. Discussion among class members will be a powerful tool for this lesson.

TEACHING THE LESSON

PART I: A. Use TR #1 to share with students a variety of everyday experiences of estimation. Can students think of other examples? Feel free to discuss other examples of estimation you or your students have encountered. As you go through each example, ask students to quickly estimate the answer. Emphasize the need for quickness and mental calculations because in most of these situations quickness is important as paper and pencil are not accessible. As students produce estimates, discuss different strategies used . At this point the strategies used may be rather crude or limited to only rounding.

Stress that there are many ways to estimate each of the examples, and throughout the year you'll be sharing a variety of them with the class. It is very important that students recognize different estimation strategies can be successful. Emphasize that a variety of strategies are acceptable if they produce a reasonable answer.

Ask students to think of examples of estimation other than those shown on TR #1. For example: Why is estimation important when using a calculator?

B. Summarize these key features of estimation:

DONE QUICKLY - as time is usually limited

DONE MENTALLY - as paper and pencil are generally not accessible

PRODUCES A REASONABLE ANSWER - as this is all the situation calls for.

Ask students to keep these features in mind. This will help students better understand the value of estimation as well as the strategies and techniques they will be learning.

PART II: A. Point out to students that they are going to learn one technique for estimating today and will be focusing on others as the year continues.

The front-end strategy can be illustrated to students by using TR #2.

The key features of the front-end strategy are:

1) focusing on the lead digits

2) adjusting the figure based on the extra information (extra digits).

Practice this technique using the "TRY THESE" exercises at the bottom TR #2. Encourage oral discussion on each of the exercises so it is clear to everyone how the the front-end strategy is used.

B. TR #3 provides an opportunity to use the front-end strategy for problems presented horizontally. Emphasize the importance of correct place value.

Also focus on only important digits to apply the front-end strategy. For example, recognizing that 68 and 89 contribute little toward making a front-end estimate is a key concept. Encourage students to take a global look at the numbers in a problem, then select appropriate values to make an estimate. This approach not only produces a good estimate, but is quicker and easier than dealing indiscriminately with all values.

Go through the "TRY THESE" exercises on TR #3 with the class. Point out features (such as unequal number of digits) that might cause students difficulty.

USING THE EXERCISES

After you have gone through the "TRY THESE" on the transparencies and discussed them, your students should be ready for more independent work. Assign student sheets 1 and 2 to be done during the remainder of the class or as homework.

ANSWER KEY

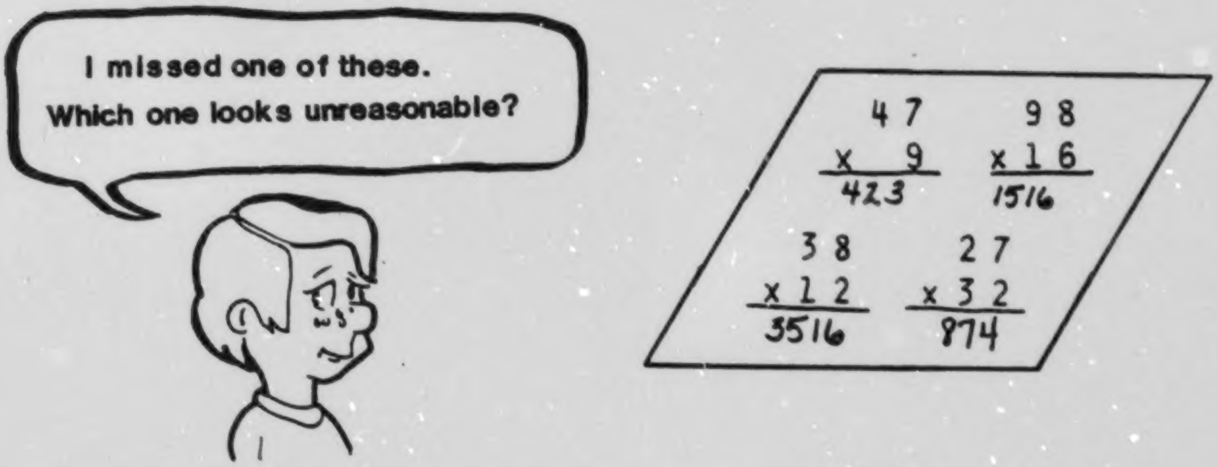
1. 31 and 119 can be ignored
2. 17 and 87 can be ignored
3. 489 and 105 can be ignored
4. 27 and 6 can be ignored
5. 81 and 197 can be ignored
6. 2298; 5027; and 1765. acceptable range: 8000-9300
7. 14,927; 33,452; and 10,253. acceptable range: 50,000-60,000
8. 1,412,537; 2,075,126; and 3,275,019. acceptable range: 6 million-7 million
9. 53,475; 10,753; and 25,127. acceptable range: 80,000-90,000
10. 152,746; 457,279; and 298,275. acceptable range: 750,000-1,050,000
11. 11; 1100; about 120 more; 1200-1300
12. 9; 9000; about 1,300 more; 10,300-10,500
13. 11; 110,000; about 11,000 more; 122,000-125,000
14. 4; 4000; about 1,200 more; 5,200-5,500
15. 6; 6000; about 1,800 more; 7,800-8,000
16. 7; 70,000; about 7,000 more; 77,000-80,000
17. 6,000-7,100
18. 15,000-16,200

19. 1,400-1,500

20. 25-35

21. (a) (answers may vary) Ashland and Iatan and Tipton
    (b) (answers may vary) Kansas City, Springfield and St. Joseph
    (c) acceptable range: 800,000-900,000

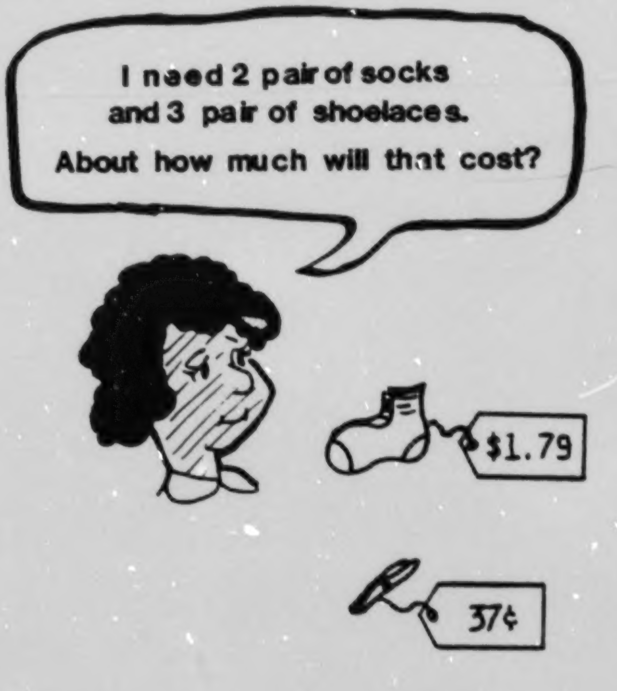
22. (a) (answers may vary) Delaware and Rhode Island
    (b) (answers may vary) Connecticut, Maine, Massachusetts, New York
    (c) 85,000-105,000 mi^2

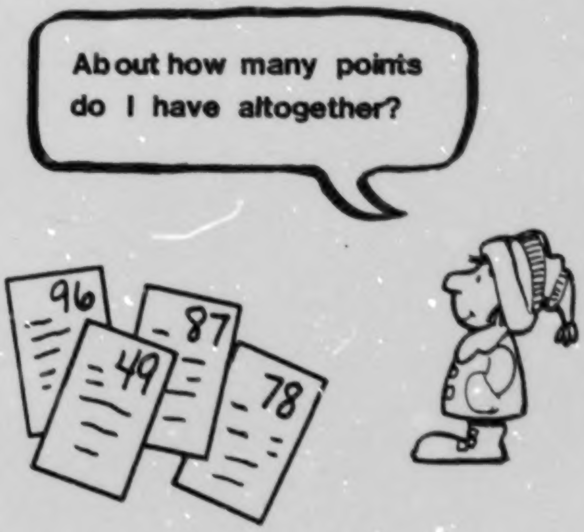

I missed one of these.
Which one looks unreasonable?
47 × 9 = 423
98 × 16 = 1516
38 × 12 = 3516
27 × 32 = 874

E.T.
THE EXTRA-TERRESTRIAL

The movie E.T. brought in a record $102 million in its first month. About how much money did it make per day?

The Thompson's dinner bill totaled $28.75. They plan to leave about a 15% tip. About how much should they leave?

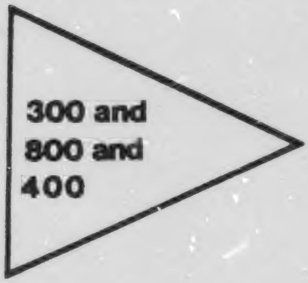

I need 2 pair of socks and 3 pair of shoelaces. About how much will that cost?
$1.79
37¢


About how many points do I have altogether?
96
87
49
78

★ Soccer Game Attendance ★

Sunday	4,312
Tuesday	1,816
Friday	2,498

Estimate the total attendance.



Front - end ▷	the lead digits total 7
Place value ▷	7 thousand
Adjust ▷	1500 more
Estimate ▷	8,500

TRY THESE:

$$\begin{array}{r} 4{,}916 \\ 2{,}392 \\ +\ 3{,}105 \\ \hline \end{array} \qquad \begin{array}{r} 1{,}716 \\ 2{,}453 \\ 3{,}312 \\ +\ 2{,}569 \\ \hline \end{array} \qquad \begin{array}{r} 24{,}316 \\ 17{,}393 \\ +\ 40{,}216 \\ \hline \end{array} \qquad \begin{array}{r} 20{,}298 \\ 4{,}316 \\ +\ 31{,}986 \\ \hline \end{array}$$

C

To Estimate:

32,412 + 1,068 + 45,089

Think:

1. Front-end ...... The most important digits are 32,412 and 45,089 7

2. Place value 70,000

3. Adjust .... The next group of important digits are 2 thousand, 1 thousand and 5 thousand. 8,000

Final estimate 78,000

****

TRY THESE:

1. 3,426 + 2,819 + 1,146

2. 23,463 + 10,416 + 1,892

3. 345,216 + 41,842 + 10,463

4. 42,463 + 1,459 + 2,563 + 30,468

Name: ____________________

Cross out two values you would ignore in making an estimate.

1.
4527
31
119
+ 7345

2.
527
489
17
698
+ 87

3.
42,756
489
56,177
+ 105

4.
119
27
897
+ 6

5. 4927 + 81 + 197 + 34625 + 5476

Check the three most important values you would use in making a front-end estimate. Then use them to make an estimate.

6.
3
59
2,298 ✓
5,027 ✓
+ 1,765 ✓

7.
14,927
33,452
189
58
+ 10,253

8.
1,412,537
2,075,126
5,894
12,176
+ 3,275,019

9. 53,475 + 829 + 45 + 10,753 + 25,127 __________

10. 152,746 + 1,573 + 457,279 + 458 + 5217 + 298,275 __________

Estimate each total.

11.
346
219
438
+ 249

12.
3,469
2,943
+ 4,009

13.
38,496
23,984
+ 61,049

	11.	12.	13.
FRONT-END	______	______	______
PLACE VALUE	______	______	______
ADJUST	______	______	______
ESTIMATE:	______	______	______

	14.	15.	16.
	3,486	1,046	31,416
	593	438	2,814
	+ 1,304	2,919	+ 44,168
		+ 3,576	
FRONT-END	________	________	________
PLACE VALUE	________	________	________
ADJUST	________	________	________
ESTIMATE:	________	________	________

17. 4,349 + 1,128 + 1,590 ≈ ____________

18. 4,260 + 10,830 + 1,090 ≈ ____________

19. 426 + 1,083 + 19 ≈ ____________

20. 14.26 + 10.83 + 5.98 + 2.19 ≈ ____________

21. If you were to estimate the population total for these towns in Missouri:

(a) Which populations could you ignore and still make a good estimate? ____________

(b) Which cities did you use? ____________

____

(c) What is your estimate? ____________

Ashland	859
Iatan	87
Kansas City	675,298
Springfield	119,247
St. Joseph	87,256
Tipton	1,426

22. If you were to estimate the total area for these eastern states:

(a) Which states could you ignore and still make a good estimate? ____________

____

(b) Which states did you use? ____________

____

(c) What is your estimate. ____________

Connecticut	5,009 mi^2
Delaware	2,057 mi^2
Maine	33,215 mi^2
Massachusetts	8,257 mi^2
New York	49,576 mi^2
Rhode Island	1,214 mi^2

NSF ESTIMATION PROJECT
GRADE 8 - LESSON 2

OBJECTIVES: Review the importance of estimation in our society.

Illustrate the front-end strategy using a money model.

Introduce the idea of adjusting by grouping compatible amounts.

TEACHER BACKGROUND

The FRONT-END strategy for addition is used in this lesson with a money model. The adjusting process is handled slightly differently than in Lesson 1. In Lesson 1, students used the next group of digits to adjust the initial estimate. For example,

3,426 + 2,119 + 743

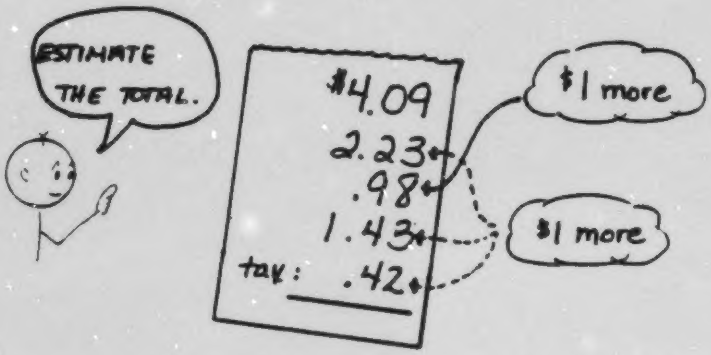
FRONT-END: 3,000 + 2,000 = 5,000

ADJUST: 400 + 100 + 700 = 1,200

ESTIMATE: 5,000 + 1,200 = 6,200

For problems containing more than 3 or 4 addends this method of adjusting, although very accurate, is too cumbersome. Instead, the adjustment is made by taking a more global look at the remaining digits and grouping them in some fashion.

For example,



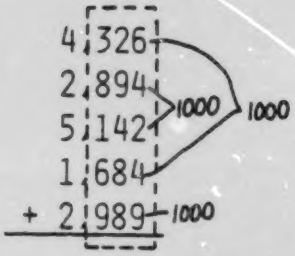
1 FRONT-END -- THE DOLLAR AMOUNTS TOTAL $7.

2 ADJUST -- THE CENT AMOUNTS TOTAL ABOUT $2 MORE.

3 ESTIMATE -- THE TOTAL IS ABOUT $9.

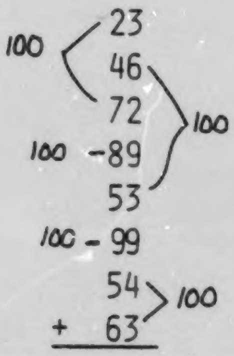
The front-end digits total $7. To adjust this initial estimate $7, the remaining digits (the cent amounts) can be grouped to dollar amounts (.98 is about $1; .23, .43, and .42 is about another $1; .09 is small so ignore it. The final estimate is about $7 + $2 or $9.

The adjustment of the initial estimate is made by grouping the unused numbers to a convenient total (in this case to dollar amounts). This same process can be used when working with whole numbers:



FRONT-END:	14
PLACE VALUE:	14,000
ADJUST:	3,000
ESTIMATE:	17,000

This adjusting by grouping compatible amounts is a powerful estimation technique which will be used in conjunction with many other estimation strategies. This process may even be used alone, without the front-end step, to form an estimate when working with smaller numbers:



ESTIMATE: ABOUT 500!

TEACHING THE LESSON

PART I. To begin this lesson, display TR #1, "That's A Fact?" Read each fact and discuss which numbers represent exact values and which are clearly estimates. Point out the use of key words (e.g. more than) for depicting estimates in some situations while in others the context alone identifies a number as an estimate. This initial discussion will help students better appreciate the uses of estimation.

An interesting and true story will help illustrate how the appearance of a number influences the message it gives to the reader. Relate this story to the class.

> A team of surveyors were sent to Mt. Everest to take an official measurement of it's height. They decided to make 6 independent height measures then average these to determine the "official" height. The 6 measurements were:
>
> The team studied the obtained average and were concerned that readers might view 29,000 ft. as a rough estimate rather than a fairly precise measure. So, they recorded their official calculation as 29,002 ft.
>
> 28,990
> 28,991
> 28,994
> 28,998
> 29,001
> 29,026
>
> 174,000 ÷ 6 = 29,000

Discuss with students why the surveyors might have been concerned with an official measure of 29,000 ft.

PART II. A. Students will be familiar with the idea of front-end estimation from Lesson 1. Display the grocery ticket on TR #2. The front-end digits total $7. Adding all the digits in the second column would be cumbersome. So rather than adjust as we did in Lesson 1, illustrate the idea of grouping the cent amounts:

$$\left.\begin{array}{r}.19\\ .86\end{array}\right\}\text{ THESE MAKE ABOUT \$1}$$

$$\left.\begin{array}{r}.39\\ .29\\ .14\\ .23\end{array}\right\}\text{ THESE MAKE ABOUT \$1}$$

The adjusted estimate then is \$7 + 1 + 1 or \$9. Provide as much practice as necessary with this "grouping" idea using the "TRY THESE" problems. Point out the difference between this adjustment by grouping and the adjustment by mentally adding digits as done in Lesson 1. When the problem involves more than 4 addends grouping will be a more useable adjusting procedure.

B. TR #3 illustrates the front-end/grouping technique using whole numbers. Allow students plenty of flexibility in what numbers they choose to group. In the example, hundreds are grouped but students in your class may suggest an alternate grouping technique.

C. Finally, TR #4 is provided to allow more practice in this grouping procedure. In this transparency, grouping is used as an entire estimation technique in itself -- the front-end approach is not needed because of the size of the numbers.

USING THE EXERCISES

Exercises are provided which give students practice using the front-end/grouping strategy on decimals (money) and whole numbers. Also provided are problems such as those on TR #4.

ANSWER KEY

1. about 200
2. about 300
3. about 3,000
4. about 3,000
5. about 200
6. about 20,000
7. about 200,000
8. about 500
9. \$4-\$5
10. \$8-\$9
11. \$10-\$11
12. \$7-\$8
13. \$14-\$16
14. \$15-\$17

15. $19-$21
16. $15-$18
17. less
18. less
19. more
20. less
21. less
22. less
23. more
24. less
25. more
26. more
27. less
28. no

That's a Fact!

Which of these facts contain estimated values and which contain exact values???

Can you tell the difference?

The Gross National Product for 1978 was $2,127,560,000,000.

The 1980 census reported an American population of more than 221 million people.

The median age of the U.S. population is 30.2 years.

Female population increased by 731,000 more than male population since 1970.

Hank Aaron hit a record total of 755 career homeruns.

Madison Square Garden in New York has a seating capacity of 19,571.

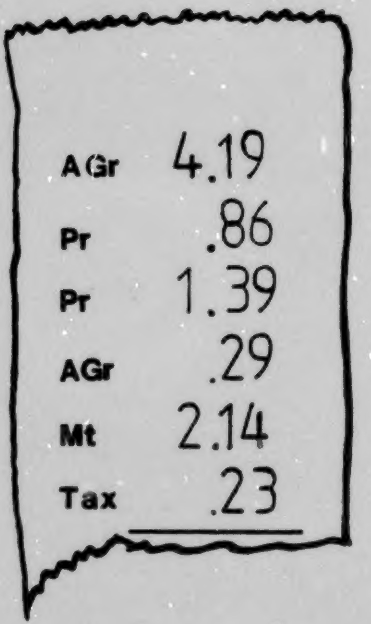
Richard Knecht, aged 18, recorded 25,222 consecutive sit-ups in 11 hours 14 minutes on Dec. 23, 1972.

The average salary of a professional basketball player is $165,000.

8-2 -TR1

This is a grocery store ticket where the amount has been torn away.

Estimate the total.



AGr	4.19
Pr	.86
Pr	1.39
AGr	.29
Mt	2.14
Tax	.23

1 **Front-end · · · The dollars total $7**

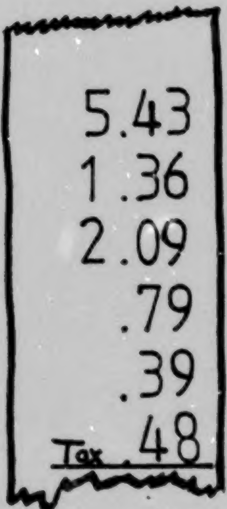
(4 + 1 + 2)

2 **Adjust · · · · · The cents are about $2**

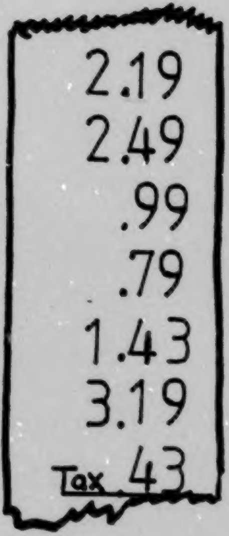
(19 and 86 are a dollar.
The rest make a dollar.)

3 **Estimate · · · · $7 + $2 = $9**

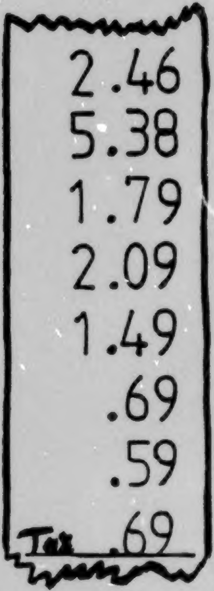
TRY THESE:



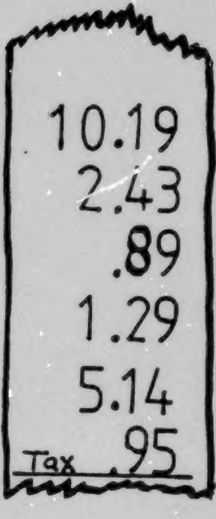
5.43
1.36
2.09
.79
.39
Tax .48



2.19
2.49
.99
.79
1.43
3.19
Tax .43



2.46
5.38
1.79
2.09
1.49
.69
.59
Tax .69



10.19
2.43
.89
1.29
5.14
Tax .95

Week	Books Sold
1	137
2	268
3	139
4	227
5	192
6	213
7	358

How many books were sold during our 7 day book drive?

Front-end ...... $\boxed{1+2+1+2+1+2+3}$ = 12
1200

Adjust (By grouping) .. 37 and 68 make 100
92 is about 100
The rest make about 100 ... 300 more

Estimate 1500

TRY THESE:

1. 4,527
1,416
3,798
+ 1,243

2. 436
129
317
486
553
+ 198

3. 3,517
1,278
2,643
3,495
+ 1,287

Front-end... ______ ______ ______

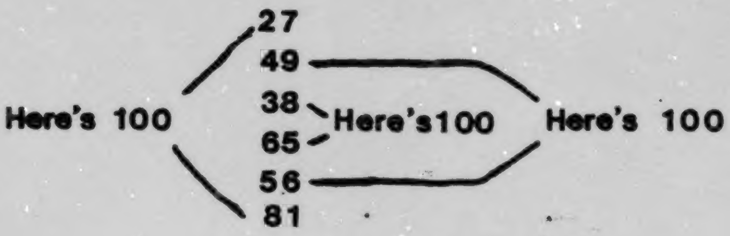
Adjust ..... ______ ______ ______

Estimate ... ______ ______ ______

This sum can be estimated by grouping numbers to form COMPATIBLE sets.

27
49
38
65
56
+ 81

Here's how: Look for hundreds!


27
49
38
65
56
81
Here's 100
Here's100
Here's 100

The sum is about 3 hundreds.

300

TRY THESE:

1. 37 + 46 + 89 + 24 + 59 + 73

2. 246 + 356 + 689 + 843 + 986 ∘∘∘∘ Look for thousands!

3.
39
46
83
72
48
59
+ 46

4.
463
549
176
328
384
246
+ 684

Name: ____________________

Check for compatible numbers as you estimate each total.

1.
72
25
89
+ 13

2.
88
55
43
78
9
+ 25

3.
615
369
812
189
575
+ 432

4.
457
539
812
641
359
+ 253

5.
12
53
85
+ 45

6.
1256
8495
2463
+ 7475

7.
51284
47592
13485
+ 85269

8.
13
246
85
+ 153

Estimate the total for each ticket.

9.
$ 1.19
.49
.79
2.13
tax .22

10.
$ 1.49
2.79
1.39
1.19
1.68
tax .41

11.
$ 4.19
3.89
1.79
.49
tax .50

12.
$ 1.49
2.39
.79
.29
1.69
.49
tax .34

13.
$ 3.46
1.59
.79
2.19
4.37
.28
1.49
tax .67

14.
$ 1.39
5.46
1.19
1.43
2.79
.46
.59
.39
1.43
tax .72

15.
$ 4.38
1.41
2.19
.89
.79
1.36
4.29
3.14
.69
tax .91

16.
$ 2.46
1.09
1.78
1.36
1.99
2.49
1.89
2.39
tax .73

Estimates for each problem below have been given. Decide whether the actual answer would be MORE or LESS than the estimate by ADJUSTING.

17.

We have collected signatures from the 3 precincts of our city. Estimate the total.

4,396
5,129
2,416

Estimate: 12,000

18.

MEMBERSHIP
DRIVE

Here are new recruits:

Mon. 39
Tues. 87
Wed. 75
Thur. 18
Fri. 129

Estimate: 400

19.

About how much will I need?

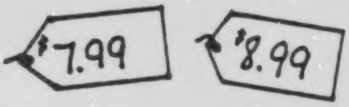
$3.14 $1.69

$2.98

Estimate: $7.00

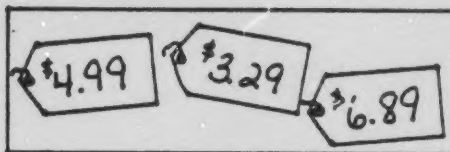
For each set of items decide whether they will total MORE or LESS than $20.

20.



more
less

21.



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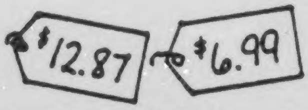

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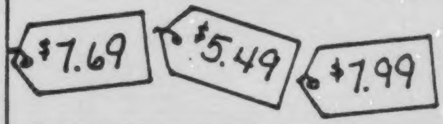

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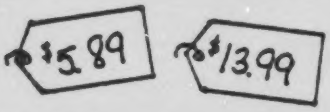
24.



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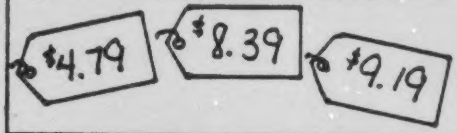
25.



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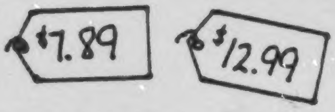
26.



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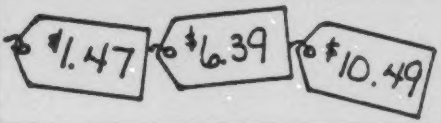
27.



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28. Estimate the answer below. Then briefly describe how you got your estimate so that it can be shared in class tomorrow.

You have only a $5 bill. Your father sends you to the store for 2 cartons of milk at $1.89 each and 3 loaves of bread at 69¢ each. Do you have enough money?	How I Did It:

NSF ESTIMATION PROJECT
GRADE 8 - LESSON 3

OBJECTIVE: To introduce and develop the clustering strategy.

TEACHER BACKGROUND

The clustering strategy is suited for a particular type of problem which is often encountered in everyday experiences. It can be used when a group of numbers cluster around a common value. (average).

Since all of the values are very close to each other in value, to estimate the total attendance for this time period, use CLUSTERING:

WORLD'S FAIR ATTENDANCE (JULY 1-6)

Mon.	72,250
Tues.	63,819
Wedn.	67,490
Thurs.	73,180
Fri.	74,918
Sat.	68,490

1. ESTIMATE AN AVERAGE

All the numbers cluster around 70,000

2. MULTIPLY THE AVERAGE BY THE TOTAL NUMBER OF VALUES

6 × 70,000 is 420,000

It is important that students recognize different practical uses of clustering. (This is explicitly done in transparencies 2 and 4.) Such experiences will not only aid their understanding of the process but also increase their sensitivity to real world applications of this powerful strategy.

The clustering strategy is only useful in a particular situation (when all of the numbers are close in value). It is powerful because it reduces a problem involving several additions to one which involves a simple multiplication. Don't worry or ask students to adjust the estimate. It is often very difficult because it is hard to determine if the average used is too high or low. If the numbers involved lend themselves to adjusting, fine but don't force an adjustment.

TEACHING THE LESSON

PART I: Display the world's fair problem on TR #1. Ask each student to estimate the sum and to record their estimate on scrap paper.

Discuss the different strategies used. Some students may use a front-end process or a rounding strategy. It is likely that someone in the class will have used the CLUSTERING idea (even before instruction). If so, have them explain the process.

Also be sure to discuss its main advantage -- reduces amount of numbers being worked with.

PART II: Use TR #2 to practice finding a quick average given a group of numbers.

Ask students to describe situations where these data might result and why totals would be needed. If necessary, get discussion rolling by asking which information would likely report:

Student test scores: About how many total points do I have?

Season attendance totals: About how many people attended during these five seasons?

Lunch bills for a group: About how much did we spend on lunch altogether?

Walking distance: About how far did I walk?

PART III: Use TR #3 to apply the clustering process.

1. ESTIMATE AN AVERAGE.

2. MULTIPLY IT BY THE NUMBER OF VALUES.

After going through the two steps --- AVERAGE - MULTIPLY --- on the daily tours example do the "TRY THESE" exercises. Discuss the estimated averages (there may be more than one used) and estimated total obtained on each of them. Use the averages obtained earlier and apply the multiplication step to obtain estimated totals.

PART IV: Show the problem in TR #4. Ask students to guess on which days there were severe thunderstorms. (Maybe the rain on Sunday and Monday kept the concert attendance figures down). Discuss why it would be better to use a clustering strategy on five days and a front-end for two than rely on a single strategy.

Encourage students to use a combination of clustering and front-end as they do the TRY THESE on TR #4. Asking several different students to discuss the procedures they used in these multistep problems will be time well spent. Also challenge them to describe situations where information like this would occur. For example,

(1)--Monthly snow fall totals from November to April--find approximate amount of snow for season.

(2)--Cost of grocery items--find total cost.

(3)--Yards gained in each of seven football games--find season total.

(4)--Population of states in a region--about how many people live in the region.

ANSWER KEY

1. Average - 60,000; Total - 300,000
2. Average - 5.00; Total - 30.00
3. Average - 30; Total - 180
4. Average - 7; Total - 42
5. Average - 8,000; Total - 72,000
6. #2
7. #1
8. #5
9. #4
10. #3
11. Average - 50,000; Total - 250,000
12. Average - 6 miles; Total - 36 miles
    You don't have to add "messy" fractions which contain unlike denominators.
13. No

14. Yes

15. No

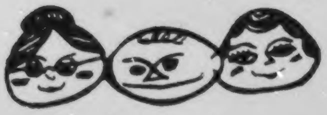
16. Yes

17. 4 x 700,000 = 2,800,000
6 x 40,000 = 240,000
About 3,000,000

18. 4 x 80,000 = $320,000
5 x 200,000 = $1,000,000
About $1.3 million

World's Fair Attendance

July 1-6



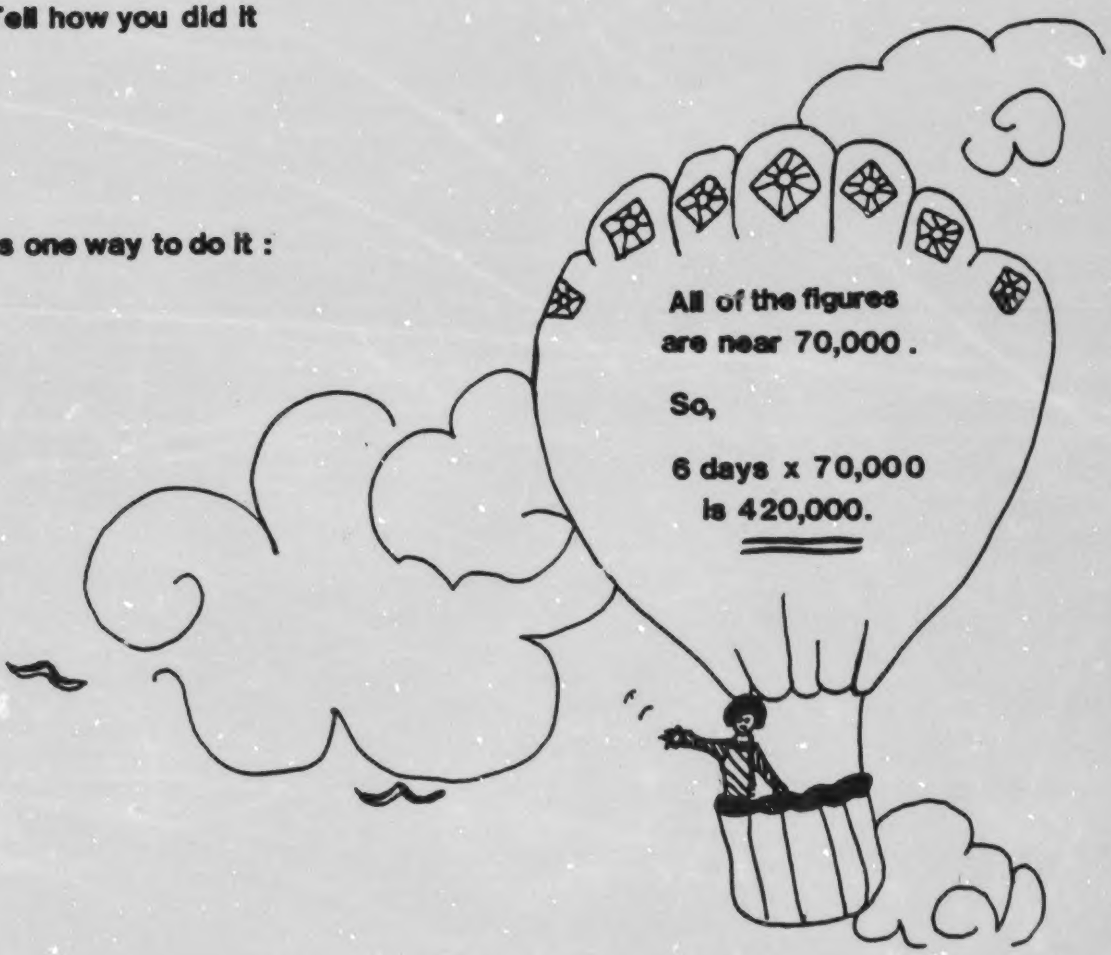
Estimate the total attendance
for this time period.

Mon.	72,250
Tues.	63,891
Wed.	67,490
Thurs.	73,180
Fri.	74,918
Sat.	68,490

* Estimate the sum

* Tell how you did it

Here's one way to do it :



All of the figures
are near 70,000.

So,

6 days x 70,000
is 420,000.

Practice Estimating An Average.

Look for a number they are all close to.

82 90 87 97 94 88 93

Average: ____

\$3.42 \$2.12 \$2.98 \$3.18 \$3.19 \$2.50

Average: ____

$3\frac{1}{8}$ $2\frac{3}{8}$ 3 $2\frac{1}{2}$ $2\frac{7}{8}$

Average: ____

$$
\begin{array}{r}
1,843,275 \\
2,134,057 \\
1,752,786 \\
2.081,927 \\
+\ 1,985,203 \\
\hline
\end{array}
$$

Average: ____

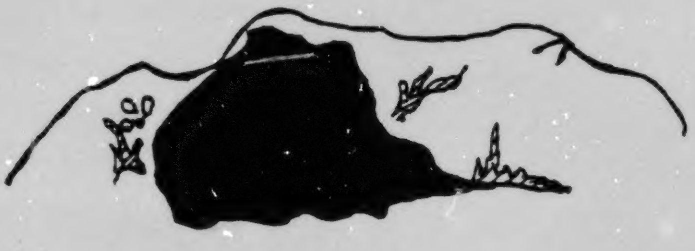
Hidden Caves
7 Tours Daily

- To estimate the total number of visitors, make a simpler problem by clustering.
- Look at all those messy numbers and choose a simple one they are all close to.

Record for July 17

Tour	Visitors
1	82
2	90
3	87
4	97
5	94
6	88
7	93

1. **Average:**
2. **Multiply:**



TRY THESE:

1. \$3.42
2.12
2.98
3.78
2.50
+ 3.79

2. $3\frac{2}{5} + 2\frac{7}{8} + 3 + 2\frac{4}{5} + 2\frac{9}{10} =$

3. 1,875 + 2,150 + 2,075 + 1,979 =

4. 104,316
98,397
81,412
+ 114,900

5. \$213,180
199,277
275,500
168,831
+ 228,362
Total:

8-3-TR3

Concert Attendance

About how many people attended the concert?

Average:

Multiply:

Front-end:

Total estimate:

Sunday	17,598
Monday	12,217
Tuesday	48,255
Wednesday	51,145
Thursday	48,285
Friday	55,285
Saturday	49,345

Use front-end here.

Use clustering here.

TRY THESE:

1.

Nov.	Dec.	Jan.	Feb.	Mar.	Apr.
$1\frac{1}{2}$	$6\frac{3}{4}$	$7\frac{1}{8}$	$6\frac{1}{9}$	$7\frac{1}{5}$	$\frac{4}{5}$

2.
.39
.42
.45
.36
3.40
+ .39

3. 28 + 35 + 29 + 28 + 32 + 200 + 25 =

4.
5,346,274
4,915,276
5,490,879
4,705,630
4,591,286
+ 11,418,251

Name ____________________

Estimate an average -- then the total.

1. 63,287
51,266
59,875
63,459
+ 64,265

Estimated

__________ Average

__________ Total

2. 4.89
5.19
5.87
4.79
4.97
+ 4.50

Estimated

__________ Average

__________ Total

3. 41
30
25
20
32
+ 29

Estimated

__________ Average

__________ Total

4. 6 3/4
7 1/8
7 5/8
6 2/3
6 9/10
+ 6 15/16

Estimated

__________ Average

__________ Total

5. 8,862
7,963
8,245
8,107
8,089
8,452
8,135
6,823
+ 8,134

Estimated

__________ Average

__________ Total

The information in Problems 1-5 could have come from different sources. Choose a reasonable match.

Problem #

6. Total cost for a group's lunches. __________
7. Total attendance for a rock concert tour. __________
8. Total cost for a fleet of cars. __________
9. Total weight of some small watermelon. __________
10. Total attendance from different scout troups. __________

11. Estimate the total attendance at last year's football games.

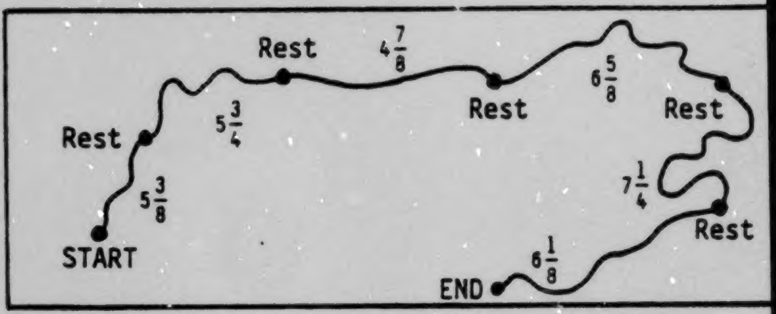
ESTIMATED AVERAGE: __________

ESTIMATED TOTAL: __________

GAME	ATTENDANCE
Army	43,941
Penn State	56,321
Colorado	42,838
Oklahoma State	48,904
Kansas	53,327

12. About how many miles long is the entire bike tour?

BIKE TOUR


START
$5\frac{3}{8}$
Rest
$5\frac{3}{4}$
Rest
$4\frac{7}{8}$
Rest
$6\frac{5}{8}$
Rest
$7\frac{1}{4}$
Rest
$6\frac{1}{8}$
END

ESTIMATED
AVERAGE: ____________

TOTAL DISTANCE: ____________

Why is clustering easier than adding all
the different fractions? ______________________________________

____

Would the clustering strategy be a good estimation strategy to use here?

13.
1.25
.023
7.843
.96
+3.52

YES NO

14.
482
512
612
409
+507

YES NO

15.
853
457
245
38
+4925

YES NO

16.
$1.5 million
2.6 million
1.92 million
1.87 million
+2.04 million

YES NO

17. Here are the salaries for a basketball team --

$750,295
710,450
687,260
675,025
50,275
42,520
41,500
37,450
35,280
33,455

About what is their total payroll? __________

Why are two averages helpful here? __________

18. Here are the prices of homes sold by one realtor this year.

$76,500
81,255
82,575
80,450
170,800
180,275
195,450
199,999
260,450

About what was the total sales for the year? __________

NSF ESTIMATION PROJECT
GRADE 8 - LESSON 4

Objectives : To estimate differences using the front-end and/or compatible numbers strategies.

TEACHER BACKGROUND :

Throughout this estimation program students should be recognizing and accepting:

- that estimation is important,
- that only an estimate is required in many situations,
- that there are many ways of obtaining an appropriate estimate, and
- that any estimate within a reasonable range is acceptable.

They also should be gaining confidence in their own ability to estimate as well as flexibility in their estimation thinking. To obtain these goals it is important that you engage them in discussion and listen to how they think. This will be especially true as this lesson is developed.

In this lesson the focus is on subtraction. Two methods for estimating differences will be explored:

1. Front-End Subtraction
2. Compatible Number Subtraction

As the size of the numbers gets larger, the emphasis will be on labeling the place value correctly. For example, to estimate 24,896 - 15,383, students will be encouraged to write 24 thousand - 15 thousand. This notation cleans up the problem by eliminating unnecessary digits and also emphasizes the correct place value. National assessments have shown that one of the major errors made by students as they estimate is incorrect assignment of place value. Whenever you are dealing with large numbers, it is a good general practice to emphasize the place value through language and in writing as in the example above.

FRONT-END SUBTRACTION

The technique of front-end estimation is extended to subtraction of whole numbers. The steps are similar to addition, although the thinking changes for Step 2.

$$\begin{array}{r} 6573 \\ -\ 3294 \\ \hline \end{array}$$

Step 1: Subtract the lead digits.	6-3=3(thousands)	
Step 2: Compare the rest.	$\begin{array}{r} 6\,5\,7\,3 \\ -\ 3\,2\,9\,4 \\ \hline \end{array}$ 573 is greater than 294.	This indicates that the answer is over 3000 since if exact computation were being performed, no borrowing would be necessary in the hundred's column.
Step 3: Adjust.	over 3000, or about 3200 or 3300	Here the front-end difference (initial estimate) of 3000 is adjusted to produce a final estimate. Over 3000 is acceptable. By subtracting the hundreds digits, or some variation of that, an estimate of 3200 or 3300 can be obtained.

If instead, the problem were:

$$\begin{array}{r} 6573 \\ -\ 3839 \\ \hline \end{array}$$

the thinking for step 2 might be:

> I can't subtract 8 hundreds from 5 hundreds so I'd need to borrow. My estimate is less than 3000.

At this point students should be able to decide if their estimated difference is more or less than the initial, front-end estimate. Some students will be able to go ahead and mentally borrow to produce an adjusted estimate of 2700.

COMPATIBLE NUMBER SUBTRACTION

Unlike addition which may contain many addends, subtraction is performed using only 2 numbers at a time. Because this is the case, frequently we can take advantage of the particular numbers involved to make our estimate. For example:

$$\begin{array}{r} 38,496 \\ -\ 19,043 \\ \hline \end{array}$$

Think:

$$\begin{array}{r} 38 \text{ thousand} \\ -\ 19 \text{ thousand} \\ \hline \end{array}$$

Although the front-end subtraction technique can be used, the data in this problem lend themselves to forming a new problem containing a compatible set of numbers:

$$\begin{array}{r} 38,000 \\ -\ 18,000 \\ \hline \end{array} \quad \text{or} \quad \begin{array}{r} 39,000 \\ -\ 19,000 \\ \hline \end{array} \quad \text{or} \quad \begin{array}{r} 40,000 \\ -\ 20,000 \\ \hline \end{array}$$

Notice that each of these new problems is relatively close to the original data but each is easier to mentally compute.

Compatible number computation simply refers to studying the given problem to see if a new problem can be formed which:

1. is relatively close to the original and
2. is easy to mentally compute.

Each of these estimation strategies will be applied to subtraction problems in this lesson. Encourage discussion as your students practice both methods.

TEACHING THE LESSON

PART I. Transparencies #1 and #2 introduce the steps of the front-end subtraction strategy. Like front-end addition, step one is simply a matter of operating (in this case, subtracting) the lead digit and assigning the correct place value to the result. This results in an initial estimate. For subtraction, the second step involves studying the next digits to see if the initial estimate should be adjusted upward or downward. Encourage students to view this step as they would if performing exact computation -- would borrowing be required?

$$\begin{array}{r} 26{,}894 \\ -\ 14{,}986 \\ \hline \end{array}$$

1. 2 - 1 = 1   10,000
2. No borrowing would be needed in the 2nd column so the estimate is more than 10,000 (10,000+)
3. 6 - 4 = 2 or 12,000 is a good final estimate.

TR #2 illustrates the opposite case where adjusting downward would be necessary. Take extra time on this example as it is more complex than TR #1.

The TRY THESE on TR #2 provide a mixture of both types of problems.

PART II. It is important for your students to realize that there are a variety of methods to estimate differences. No doubt, some of your students have already been using a more traditional rounding approach. The aim of these lessons is to present alternative strategies to students so that they can eventually make their own choices.

A powerful strategy which is a form of rounding is the COMPATIBLE NUMBERS strategy. It has already been used in a slightly different form in previous lessons. TR #3 highlights how this idea can be used to produce quick estimated differences.

For example,

63,493 - 25,649

Think

How can the problem be cleaned up to produce a more compatible set of numbers to work with?

Here are several ways:

63 thousand - 23 thousand
or
65 thousand - 25 thousand

Either of these new problems is relatively close to the orginal and yet much easier to mentally process. Encourage students to be flexible in translating the given problem to a more convenient, relatively accurate form.

PART III. The final transparency provides an opportunity to practice estimating differences when large numbers are involved. Again, the focus is on establishing the correct place value through an emphasis on written place value.

ANSWER KEY

1. a. over 200
   b. under 200
2. a. under 500
   b. over 500
3. a. under 500
   b. over 500
4. a. over 3000
   b. under 3000
5. a. under 2000
   b. over 2000
6. a. under 42,000
   b. over 42,000
7. a. under $2
   b. over $2
8. a. under $130
   b. over $130
9. over 500; 510-520
10. under 200; 130-150
11. over $2; $2.30-$2.50
12. over 3000; 3300-3500
13. under $3; $2.50-$2.70
14. over 4000; 4400-4600
15. over $3000; $3300-$3500
16. under 200; 140-190

17. under 4000; 3600-3900

18. $5-$5.50

19. $3000-3500

20. 5500-5900

21. $35,000-$38,000

22. $34,000-$35,000

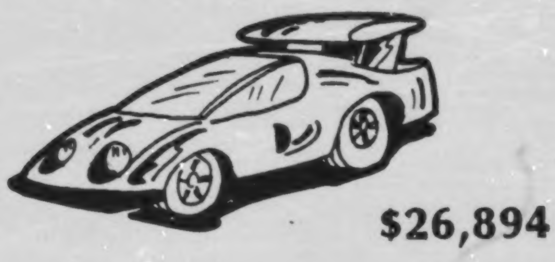
23. $23,000-$24,000

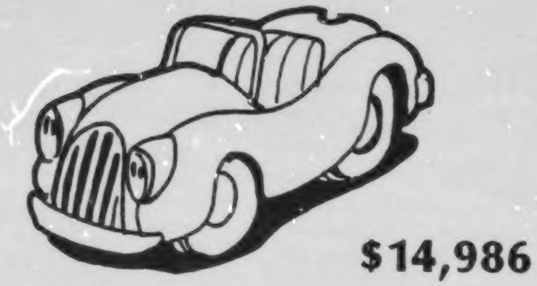
24. F

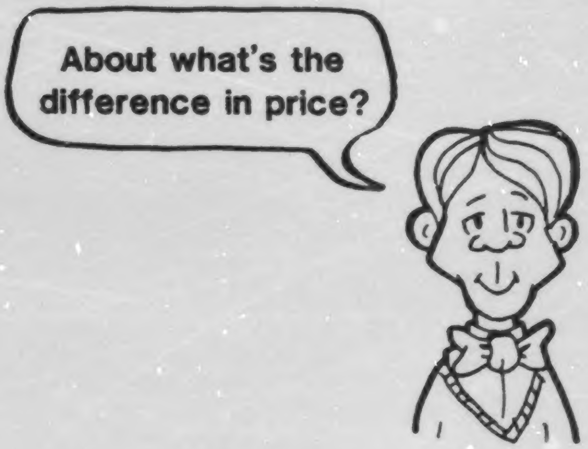
25. T

26. T

27. T






About what's the difference in price?

1. Front-end difference.

$$\begin{array}{r} 26{,}894 \\ -\ 14{,}986 \\ \hline 10{,}000 \end{array}$$

2-1 is 1
The place value is
10,000

First estimate

2. Compare the rest.

$$\begin{array}{r} 26{,}894 \\ -\ 14{,}986 \\ \hline \end{array}$$

No borrowing needed
It will be
more than 10,000

Better estimate

3. How much more?

$$\begin{array}{r} 26 \\ -\ 14 \\ \hline \end{array}$$

6-4 is 2
or 12,000

Final estimate

63,493
- 25,649

1. Front-end difference.

63,493
- 25,649
40,000

6-2 is 4
The place value is
40,000.

First estimate

2. Compare the rest.

63,493
- 25,649

Since I'd need to borrow here,
my estimate is really
less than 40,000.

Second estimate

3. How much less?

63 —
- 25 —

If I borrowed , I'd get
13-5 or 8.
So, my estimate is about 38,000.

Final estimate

TRY THESE:

74,368
- 38,499

38,496
- 15,848

4,863
- 2,946

8-4-TR2

Compatible Number Subtraction

$$\begin{array}{r} 63{,}493 \\ -\ \underline{25{,}649} \end{array}$$

Think. . . How can we change this problem just a little to make it easier to mentally compute?

$$\begin{array}{r} 63{,}000 \\ -\ \underline{23{,}000} \end{array} \quad \text{or} \quad \begin{array}{r} 65{,}000 \\ -\ \underline{25{,}000} \end{array} \quad \text{or} \quad \begin{array}{r} 63{,}000 \\ -\ \underline{25{,}000} \end{array}$$

TRY THESE:

	1.	2.	3.
	$\begin{array}{r} 26{,}483 \\ -\ \underline{9{,}683} \end{array}$	$\begin{array}{r} 4{,}546{,}009 \\ -\ \underline{2{,}943{,}046} \end{array}$	$\begin{array}{r} 46{,}893 \\ -\ \underline{27{,}416} \end{array}$
	⇓	⇓	⇓
Compatible set	26 thousand - ______	4.5 million - ______	______
	⇓	⇓	⇓
Estimate	______	______	______

Estimate:

$$\begin{array}{r} 8{,}426{,}983 \\ -\ 546{,}823 \\ \hline \end{array}$$

When estimating with large numbers, it often helps to write the approximate number using words. . .

That's about
8 million − $\frac{1}{2}$ million
or 7 $\frac{1}{2}$ million

Estimate :

$$\begin{array}{r} 23{,}467{,}432 \\ -\ 5{,}546{,}593 \\ \hline \end{array}$$

That's about
23 million − 5 million

Estimate: 18 million

TRY THESE:

$$\begin{array}{r} 9{,}234{,}981 \\ -\ 489{,}339 \\ \hline \end{array} \qquad \begin{array}{r} 12{,}776{,}324 \\ -\ 2{,}489{,}492 \\ \hline \end{array}$$

$$\begin{array}{r} 56{,}229{,}119 \\ -\ 27{,}554{,}889 \\ \hline \end{array} \qquad \begin{array}{r} 123{,}334{,}876 \\ -\ 52{,}665{,}896 \\ \hline \end{array}$$

Name: ____________________

Draw arrows to show if the estimate is UNDER or OVER the amount shown.

1. 375 − 169 | over / 200 / under | 375 − 189

2. 837 − 359 | over / 500 / under | 837 − 319

3. 642 − 155 | over / 500 / under | 682 − 155

4. 5147 − 2088 | over / 3000 / under | 5147 − 2488

5. 6327 − 4589 | over / 2000 / under | 6327 − 4189

6. 85,291 − 43,756 | over / 42,000 / under | 85,291 − 43,056

7. 14.69 − 12.75 | over / $2.00 / under | 14.69 − 12.35

8. $186.54 − 59.63 | over / $130 / under | $186.54 − 55.63

Use front-end or compatible numbers to estimate each of these.

9. 852 − 335

[✓ over]
500
under

About ________

10. 427 − 286

over
200
under

About ________

11. $8.61 − 6.24

over
$2.00
under

About ________

12. 8621
- 5264

over
3000
under

About ________

13. $5.28
- 2.64

over
$3.00
under

About ________

14. 9624
- 5187

over
4000
under

About ________

---

15. $9624
- 6210

over
$3000
under

About ________

16. 841 - 697

over
200
under

About ________

17. 5694
- 1879

over
4000
under

About ________

---

18. $9.17
- 3.86

About ________

19. $8111
- 4888

About ________

20. 8613 - 2984

About ________

---

21. $45,106
- 8,727

About ________

22. $50,000
- 15,325

About ________

23. $34,471
- 10,609

About ________

The 5 largest countries in Europe:

France	211,207 sq. miles
Spain	194,896 sq. miles
Sweden	173,731 sq. miles
Finland	130,119 sq. miles
Norway	125,181 sq. miles

Use this information to decide if these sentences are TRUE or FALSE.

_____ 24. France is more than 100,000 sq. miles larger than Norway.

_____ 25. Sweden is about 21,000 sq. miles smaller than Spain.

_____ 26. Finland is less than 5000 sq. miles larger than Norway.

_____ 27. Sweden is more than 40,000 sq. miles greater than Finland.

NSF ESTIMATION PROJECT
GRADE 8 - LESSON 5

OBJECTIVES : To illustrate and practice estimation of products with whole numbers using a rounding strategy.

To determine the direction of adjustment as a natural companion of rounding.

TEACHER BACKGROUND

This lesson uses traditional rounding techniques to estimate products of whole numbers. Special rounding techniques and mental computation involving multiples of ten should have been reviewed prior to this lesson. If students are weak in either rounding or mental computation, you may need to spend additional time on those skills.

Traditional work with rounding usually results in questions, such as: "Should I round to tens, hundreds or thousands?" Without direction from you or a natural understanding of estimation it is difficult to decide what to round a number to. In real life situations, the numbers themselves, along with the problem context, hold the key. For example,

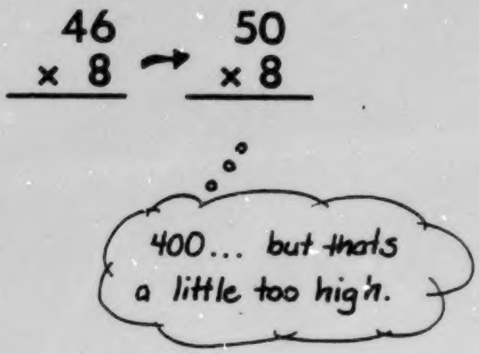
> If the attendance was 49,269 at the game and someone asked, "How many people attended the game?", it would seem natural to say either "49,000 or "about 50,000." Reporting the exact value or the nearest tens or hundreds would interest no one except the ticket sellers!
>
> If you wanted to know the population of Mexico City, an answer of 14 million would probably be more useful than 13,993,866.
>
> If the price of a shirt is $8.88 and someone asked, "How much is that shirt?", it would seem natural to say "$9."

In each of these cases, the rounding procedure was a convenient way of cleaning up numbers so they are easy to handle. Since nearly all of our estimation strategies rely heavily on mental computation, we round numbers to values that are reasonable, yet easy to compute. This is a powerful and extremely useful rule that is also very flexible. It will be followed in all of our estimation lessons.

The majority of this lesson is devoted to multiplication involving two or more digits. We want students to sense that estimation is a powerful tool and feel this is more likely to happen when large and/or messy numbers are involved.

When a one by two digit problem is to be estimated, we recommend that the one digit factor be unchanged, while the other factor is rounded to an easy number to compute.

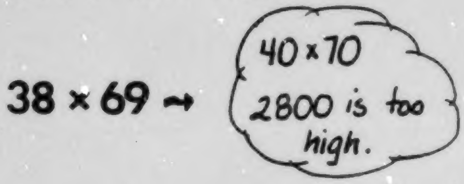
Adjustment : At this stage we want students to realize when an estimate is too high or too low. Stronger students may be encouraged to actually make some adjustment for the rounding error. In this example, the caption could be extended to "400, but this is too high. So I'll say about 350."


46 × 8 → 50 × 8
400... but thats a little too high.

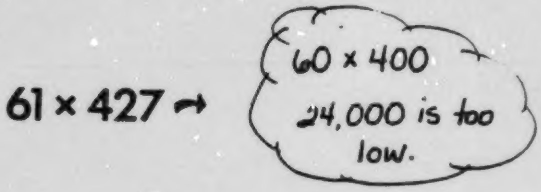
This adjustment should be a natural process, often more of a gut level feeling than an algorithmic procedure.

When both factors have two or more digits, each will be rounded to values easy to compute and this provides the heart of this lesson.

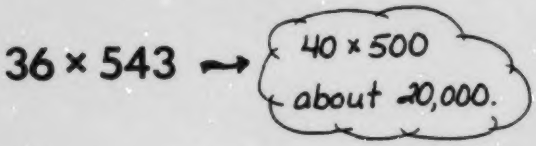
If both factors are rounded up, students should recognize the product is too high. The adjusted estimate could be written 2800- indicating that the estimate should be less than 2800.



Likewise, when both factors are rounded down, students should recognize the product is too low. So a refined estimate is 24000+.



If one factor is rounded up and the other down, there is no general rule for quickly telling whether the product is too high or low.



Good estimators have been found to employ modifications of the technique, especially when both factors are 2-digit numbers. Three of them are shown.

a. 19 x 43 : Round one factor to tens: 20 x 43
Then do 20 x 43 mentally: 20 x 43 = 860

b. 28 x 48 : Round one factor to tens
and the other factor to fives: 25 x 50
Then do 25 x 50 = 1250

c. 35 x 65 : When both factors end in 5, 40 x 60 = 2400
round one up and the other or
down: 30 x 70 = 2100

Be alert to these and other variations that some individuals may use. Encourage them to share their thinking with the other students. Remember that any estimate is acceptable as long as it falls within a reasonable range of the exact answer.

TEACHING THE LESSON

PART I: Use TR #1 to establish the three step rounding procedure:

1. ROUND -- to numbers that are easy to mentally compute
2. MULTIPLY -- mentally compute the rounded digits
3. ADJUST -- whenever factors have been rounded in the same direction, decide how the estimate should be adjusted

Point out during your discussion of the "TRY THESE" that #3 illustrates that one digit factors need not be rounded. In fact some students may mentally compute an exact answer. If it happens fine, but an estimate 490 or "a little less" is equally fine.

PART II: Tr #2 and #3 provide additional practice in using the rounding strategy and suggest additional adjustment techniques.

TR #2 illustrates rounding to various precisions. Since students should feel comfortable with different estimates, please emphasize that from here on, how they round is up to them. They should only round as closely as they can handle mentally, each to their own ability. This freedom of choice should be reflected throughout the discussion of the "TRY THESE."

TR #3 helps identify bounds for larger products. Mention that most people need almost one minute to find the exact products whereas a good estimate can be made in seconds. Students should realize that the same principles for rounding and finding estimated products apply for large numbers. It might impress some students to know they can estimate a product (such as #2 on TR #3) that overloads most inexpensive hand calculators. Problem #2 also allows for several combinations of answers and should promote discussion. Help students recognize that correct answers (i.e. too high, low, hard to tell) will vary and depend on the strategy used. You might discuss the difference between estimates produced with 2,000 x 80,000 and 2,000 x 83,000 for the example on the transparency. Which is better? Is 83,000 a mentally managable factor?

Although adjusting answers is not a central focus of this lesson, some feeling for how an estimate compares to the exact answer should be fostered. Students should recognize an estimate as a ceiling when all factors are rounded down. When some factors are rounded up and others down, the estimated product will usually be close to the exact value but it is often very difficult to determine whether it is too high or low. Encourage students not to spend a lot of time trying to make these comparisons. After all, if you are going to spend very much time on it, you might as well compute an exact ANSWER.

ANSWER KEY

1. 560
2. 5600
3. 5600
4. 2000
5. 200
6. 2000
7. 2100
8. 2100
9. 2100
10. 21,000
11. 2,100,000

12. 24,000
13. 90,000
14. 40 x 40
15. 80 x 30
16. 200 x 70
17. 20 x 90

Answers may vary on 18-27

18. 90 x 70; 6300-
19. 90 x 40; 3600-
20. 70 x 40; 2800+
21. 60 x 40; 2400+
22. 70 x 50; 3500; no adjustment or ?
23. 500 x 60; 30,000-
24. 20 x 300; 6,000; no adjustment or ?
25. 300 x 400; 120,000; no adjustment or ?
26. 100 x 100; 10,000-
27. 800 x 90; 72,000-
28. $10 x 50 = $500
29. 40 x 200 = 8,000
30. 50 x 200 = 10,000
31. 80 x 50 = 4,000 or 90 x 50 = 4,500
32. 80 x 400 = 32,000 or 90 x 400 = 36,000
33. 20; 1200; 72,000; 1,440,000; 42,000,000; 420,000,000
34. 2 4
    3 4
    3 9
35. either 3 or 4
36. either 4 or 5

48 p. box
Good BAR

There are 48 candy bars in a box. 57 boxes were ordered for the school sale. About how many candy bars is that?

To Estimate:

	Round	Multiply	Adjust
48 x 57	50 x 60	50 x 60 3000	less than 3000

rounded up so, 3000

To Estimate:

84 x 21	Round	20 x 80
	Multiply	1600
	Adjust	more than 1600 or 1600⁺

TRY THESE:

1. 79 x 28	2. 83 x 71	3. 68 x 7	4. 42 x 33
Round _____	Round _____	Round _____	Round _____
Multiply _____	Multiply _____	Multiply _____	Multiply _____
Adjust _____	Adjust _____	Adjust _____	Adjust _____

How do you decide what to round to when estimating?

That depends on:

1. The specific numbers in the problem.
2. How well you can multiply mentally.

Example:

21 x 169 = (is approximately)

20 x 200 = 4000 → Good estimate

20 x 170 = 3400 → Better estimate

21 x 200 = 4200 → Also a good estimate

21 x 170 = 3570 → Great estimate, but who wants to multiply 21 x 17 mentally?

TRY THESE:

Round, then estimate.

1. 37 x 109 =
2. 18 x 312 =
3. 11 x 514 =

Estimate:

$$\begin{array}{r} 67413 \\ \times\ 586 \\ \hline \end{array}$$

Round

$$\begin{array}{r} 7\underline{0},\underline{000} \\ \times\ 6\underline{00} \\ \hline \end{array}$$

Multiply 42,$\underline{000}$,$\underline{000}$

Adjust lower than 42,000,000
or
42,000,000⁻

Both factors were rounded up, so need to take some off.

To Estimate:

$$\begin{array}{r} 83,149 \\ \times\ 1927 \\ \hline \end{array}$$

	Up		Down
Round	2$\underline{000}$	x	83,$\underline{000}$
Multiply	166,$\underline{000}$,$\underline{000}$		

How would you adjust?

+
?
-

Rounded one factor up and one down, so it is hard to tell.

TRY THESE:

1. $\begin{array}{r} 42,389 \\ \times\ 637 \\ \hline \end{array}$

2. $\begin{array}{r} 77,469 \\ \times\ 49 \\ \hline \end{array}$

3. $\begin{array}{r} 186,438 \\ \times\ 34 \\ \hline \end{array}$

	1.	2.	3.
	+	+	+
How would you adjust?	?	?	?
	-	-	-

Name ____________________

Use mental computation to find the products.

1. 70 x 8 = ________

2. 70 x 80 = ________

3. 7 x 800 = ________

4. 40 x 50 = ________

5. 4 x 50 = ________

6. 4 x 500 = ________

7. 30 x 70 = ________

8. 3 x 700 = ________

9. 7 x 300 = ________

10. 300 x 70 = ________

11. 3,000 x 700 = ________

12. 20 x 30 x 40 = ________

13. 30 x 50 x 60 = ________

DO THIS FIRST

Circle what you would use to estimate if you round both factors.

14.	36 x 42	30 x 40	30 x 50	40 x 40	40 x 50
15.	82 x 27	80 x 20	80 x 30	90 x 20	90 x 30
16.	212 x 71	200 x 70	200 x 80	300 x 70	300 x 80
17.	24 x 89	20 x 80	20 x 90	30 x 80	30 x 90

Estimate. Then decide how your estimate should be adjusted.

18. 86 x 67 ________ + ? -

19. 87 x 36 ________ + ? -

20. 71 x 43 ________ + ? -

21. 63 x 44 ________ + ? -

22. 71 x 48 ________ + ? -

23. 485 x 59 ________ + ? -

24. 18 x 315 ________ + ? -

25. 293 x 408 ________ + ? -

26. 98 x 99 ________ + ? -

27. 777 x 88 ________ + ? -

Estimate each answer by rounding.

28. Jim earns $9.40 a week doing chores. How much is this in one year?

29. A baseball umpire works home plate 41 times a season. At an average of 194 pitches per game, how many pitches per season does an umpire call?

30. If the average junior high student answers 53 questions (oral and written) per school day, find the total for a 179 day school year.

31. The Jones Medical Center averages 83 births a week. What is the total for one year?

32. Chris has 82 regular customers on her paper route. About how many papers does she deliver each year.

33. There are 19 pairs of jeans sold every second in the United States. Use the table below to determine about how many are sold in 1 year.

TIME	**1 SECOND**	**1 MINUTE**	**1 HOUR**	**1 DAY**	**1 MONTH**	**1 YEAR**
NUMBER OF JEANS	20					

THINK
IT
THROUGH

34. How many digits does each product contain? (You don't have to write the estimated product.)

PROBLEM	NO. OF DIGITS	PROBLEM	NO. OF DIGITS
3 x 28		43 x 68	
3 x 89		4 x 743	
43 x 19		8,342 x 36,475	

35. If you multiply a 2 digit number by a 2-digit number, how many digits will the product have?

36. If you multiply a 2-digit number by a 3-digit number, how many digits will the product have?

NSF ESTIMATION PROJECT
GRADE 8 - LESSON 6

OBJECTIVE: To estimate whole number products using the FRONT-END strategy.

TEACHER BACKGROUND

This lesson describes the FRONT-END strategy for multiplication. Just as in front-end addition, the lead digits are the focus. Here is an illustration of the strategy:

$$8 \times \$4.56$$

1. Multiply the lead digits — 8 x 4.00 is $32.00
2. Estimate the rest — 56 is about 60 8 x 60 is $4.80
3. Adjust ➞

$$\begin{array}{r} 32.00 \\ +\ 4.80 \\ \hline \$36.80 \end{array}$$

This strategy is powerful in that it has the adjust step built in and generally yields a more accurate estimate than rounding. The front-end strategy is most useful for problems with a 1-digit factor. At first, students may need to record intermediate values as they use the front-end strategy. If so, encourage them to do so.

Students should realize that written ROUNDING or FRONT-END can be used to estimate multiplication. It is their decision but is often influenced by the numbers involved. For example, Front-End is usually easier and more productive than rounding when a one digit and another multi-digit factor are involved.

$$\left.\begin{array}{c} 4 \times 279 \\ \text{or} \\ 8 \times 5287 \end{array}\right\} \text{Lend themselves to Front-End}$$

If each factor has several digits, such as these, rounding is generally quicker and easier.

47 x 29 } Lend themselves
3156 x 895 } to rounding.

Each strategy produces a reasonable estimate. Once the strategies are understood and mastered, the choice of which to use in a given situation rests with the individual.

TEACHING THE LESSON

PART I. Use TR #1 to help students understand and appreciate the uses of both FRONT-END and BACK-END computation. A BACK-END process is generally used when we find an exact product. The FRONT-END process is all that is needed to produce a good estimate.

PART II. Use TR #2 to develop the FRONT-END strategy for problems with a 1-digit factor.

1. Multiply using the lead digit. Is $12.00 an over- or under-estimate? To get a closer estimate, we need to look at the rest of the digits.
2. We have rounded 78 to 80, but you could also simply use 70 --- this will let you see the digit being multiplied.
3. Adding the results from steps 1 and 2 gives us our ADJUSTED estimate.

Students should practice this procedure on the "TRY THESE," which also includes several involving whole numbers.

TR #3 extends the FRONT-END one digit multiplication to larger numbers. The emphasis should be on getting a reasonable answer quickly as students work the "TRY THESE."

ANSWER KEY

1. 2400
2. 7200
3. 3500
4. 1800
5. 2580
6. 7380
7. 3920
8. 1980

9. $6
90¢
$6.90

10. $120.00
3.00
$123.00

11. $48.00 or $48.00
1.60 2.00
$49.60 $50.00

12. 4800
300
5100

13. 350,000
25,000
375,000

14. 320,000
24,000
344,000

15. 10,000
2,000
12,000

16. 1,400,000
280,000
1,680,000

17. 54,000,000
1,800,000
55,800,000

18. 3540 x 4

12,000
2,000
14,000

19. 6 x 764

4200
360
4560

20. 6 x 47,752

240,000
48,000
288,000

21. 7 x 105,561

700,000
42,000
742,000

22. 8 x 2.39

$16.00
3.20
$19.20

To Find An Exact Answer....

Multiplying usually starts at the "back-end" of the problem:

$$\begin{array}{r} \longleftarrow \\ 5 \\ 437 \\ \underline{\times\ 8} \\ 6 \end{array}$$

7 x 8 is 56

We work our way to the front:

$$\begin{array}{r} 2\,5 \\ 437 \\ \underline{\times\ 8} \\ \underleftarrow{3496} \end{array}$$

To Find A Good Estimate....

Multiplying starts at the front-end :

$$\begin{array}{r} \overrightarrow{\underline{4}37} \\ \underline{\times\ \underline{8}} \\ 3200 \end{array}$$

400 x 8

$\boxed{3200^{+}}$

Good estimate

240

30 x 8

$\boxed{3440^{+}}$

Better estimate

In estimating, use as many digits as will give you the kind of estimate you need.

Pro Tennis Balls: $3.78

About how much will 4 cans cost?

1

Multiply the lead digit.

4 x 3.78

4 x $3.00 ⟶ $12.00

2

Estimate the rest.

78 is about 80

4 x 80 ⟶ $3.20

3

Adjust

1200 + 320 = $15.20

TRY THESE:

7 x $7.41

5 x $6.19

4 x$3.68

9 x 324

5 x 5163

Front-end Multiplication

(Big numbers)

At $8 a ticket, about how much money was brought in by the concert?

Seating Capacity:
4,625

1. Multiply by the lead digit

   8 x 4,625 = $32,000

2. Multiply by the next digit

   8 x 625 = 4800

3. Adjust

   32,000 + 4,800 = $36,800

TRY THESE:

5 x 8,345 =

1. Lead digit
2. Next digit
3. Estimate

4 x 62,425 =

1. Lead digit
2. Next digit
3. Estimate

6 x 148,785 =

1. Lead digit
2. Next digit
3. Estimate

8-6-TR3

NAME ______________________________

Some of each of these problems have been blacked out.
Use the information given to find an estimate for each.

1. 4[redacted] x 6

2. 8[redacted] x 9

3. 5[redacted] x 7

4. 3[redacted] x 6

Find a closer estimate.

5. 43[redacted] x 6

6. 82[redacted] x 9

7. 56[redacted] x 7

8. 33[redacted] x 6

Use FRONT-END to find an estimate: Remember the steps ---

1 lead digit product
2 estimate the rest
3 adjust

9. $2.29 x 3
1 $6 ← 3x$2
2 90¢ ← 3x30¢
3 $6.90

10. $20.49 x 6
1 ________
2 ________
3 ________

11. $6.24 x 8
1 ________
2 ________
3 ________

12. 853 x 6
1 ________
2 ________
3 ________

13. 74,841 x 5
1 ________
2 ________
3 ________

14. 43,251 x 8
1 ________
2 ________
3 ________

15. 2,387 x 5
1 ________
2 ________
3 ________

16. 237,421 x 7
1 ________
2 ________
3 ________

17. 6,204,752 x 9
1 ________
2 ________
3 ________

Apply your estimation skills here.

18. About 3540 calories must be burned to reduce one pound of body weight. About how many calories will need to be burned to lose 4 pounds?

____

19. During 1979 the U.S. Immigration Service reported 764 immigrants from Sweden and about six times as many immigrants from the USSR. About how many immigrants were reported from Russia?

____

20. The Department of Defense reported 47,752 U.S. casualties in the Vietnam War. About 6 times as many U.S. casualties were reported in World War II. About how many U.S. casualties resulted during World War II?

____

21. The National Center for Health Statistics reports that 105,561 accidental deaths were reported in 1978. Nearly 7 times as many cancer related deaths were reported. About how many cancer related deaths occurred in 1978?

____

22. It is estimated that each person in the United States drinks about 8 packs of soda a year. If each 6-pack costs about $2.39, about how much does an average person spend per year?

____

NSF ESTIMATION PROJECT
GRADE 8 - LESSON 7

OBJECTIVES : Estimate quotients by finding the correct place value and lead digit.

TEACHER BACKGROUND

Long division is a difficult and time consuming operation. No computation topic produces more errors than division, so estimation skill takes on increasing importance. The value of estimation strategies is to produce a reasonable answer quickly, which is the purpose of this discussion.

1. Finding the correct place value. This is an important step in estimation. Once completed it provides some definite bounds on the quotient. Here are two different approaches that locate the place value of the quotient.

A.

$$8\overline{)4\,1\,8\,7}$$

$$8\overline{)\underline{4}\,1\,8\,7}$$

Look at the thousands.

4 thousands -- there are not enough thousands to divide.

$$8\overline{)\underline{4\,1}\,8\,7}$$

Look at the thousands and hundreds.

41 hundreds -- you can divide 8 into 41.

$$\begin{array}{r} \_\ \_\ \_ \\ 8\overline{)4\,1\,8\,7} \end{array}$$

The first digit in the quotient must be hundreds -- so there must be 3 digits in the quotient.

Placing the dashes in the problem (either mentally or in writing) provides a reminder that the quotient must have three digits. Thus it must be between 100 and 999. This is an algorithmic technique but highly useful in division.

Another approach relies on multiplication by powers of ten. For example:

B. $8\overline{)4187}$

Since 4187 is greater than 800, the quotient must be greater than 100.

$$8 \times 100 = 800$$

So the quotient is greater than 100 but less than 1000. It must be in the hundreds!

Since 4187 is less than 8000, the quotient must be less than 1000.

$$8 \times 1000 = 8000$$

This approach rewards mental arithmetic and develops a good quantitative sense. Students adept at multiplication by powers of ten can quickly obtain place value bounds on any quotient.

2. Finding the lead digit. To find the first digit in the quotient divide 8 into 41 as if you were performing exact computation. This may not always be the closest number of hundreds but will always be a lower bound. Furthermore, it provides a good estimate for the quotient.

$$\begin{array}{r} \underline{5\ \_\ \_} \\ 8\overline{)4\ 1\ 8\ 7} \end{array}$$

So my estimate is 500.

Together these two steps provide an easy strategy for estimating the quotient.

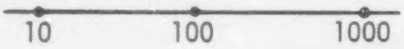
1. Identify the number of digits.
2. Find the lead digit.

There are certainly other approaches to estimating quotients (compatible number division will be introduced in Lesson 8) but this front-end strategy provides an easy to learn technique which complements exact computation instruction and also provides a method for finding a good estimated quotient.

TEACHING THE LESSON

Part I. Develop the two steps shown on TR #1. In Step 1, encourage students to find the number of digits by sliding their finger along until the number of digits in the quotient is clear. Placing a hash mark immediately above the appropriate position provides a useful record. As soon as they decide the number of digits (Step 1) encourage them to determine the lead digit (Step 2). Be sure students realize that this procedure produces a good estimate.

Part II. TR #2 encourages students to do the mental multiplication so they can bound the quotient. Ask them to place a check on the continuum but don't push for too much accuracy. You might simply ask if it is closer to one or the other. You may mention that the scale on the continuum varies but don't make a big deal of it.



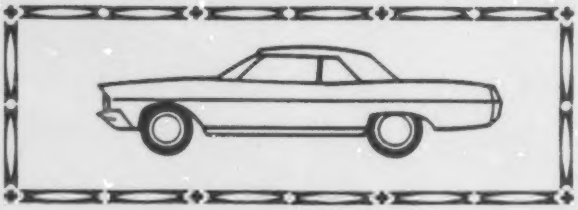
Part III. TR #3 allows students to make an estimate. The choice of method is theirs. Encourage discussion of the different strategies used. Also ask them to make up a problem that fits the data in the TRY THESE problems. (About how much does Bill Ball make per week? per year?)

ANSWER KEY

1. 3
2. 2
3. 3
4. 4
5. 4
6. 3
7. 5
8. 4
9. 4
10. 4
11. 100
12. 2,000
13. 300
14. 5,000
15. 400
16. 100
17. 5,000
18. 3,000
19. 1,000
20. 80,000
21. 400
22. 1,000
23. 10,000
24. 500
25. 600
26. 600
27. 6,000
28. 100,000
29. 400
30. 7,000
31. 2,000
32. 10,000

A car cost $9750.

About what will each of the 48 payments be?



- **Are there enough thousands?** NO $\begin{array}{r} ? \\ 48\overline{)\,9750} \end{array}$

1 Find number of digits.

- **Are there enough hundreds?** YES $\begin{array}{r} ?\,\text{- - -} \\ 48\overline{)\,9750} \end{array}$

- **What is the first digit?** $\begin{array}{r} 2\,\text{- - -} \\ 48\overline{)\,9750} \end{array}$

2 Decide on first digit.

My estimate is over 200.

TRY THESE:

$8\overline{)\,26594}$

$12\overline{)\,37523}$

$389\overline{)\,842757}$

Where does the first digit go?

How many digits in the quotient?

What is the first digit?

Total cost of tuition and books at my college is $2645 a year.

About how much is that per month?

1

It's in the hundreds.

$10 $100 $1000

2

12 x 10 = 120
12 x 100 = 1200
12 x 1000 = 12000

so

$$\begin{array}{r} 200 \\ 12\overline{)2645} \end{array}$$

over $200

TRY THESE:

$8\overline{)5555}$

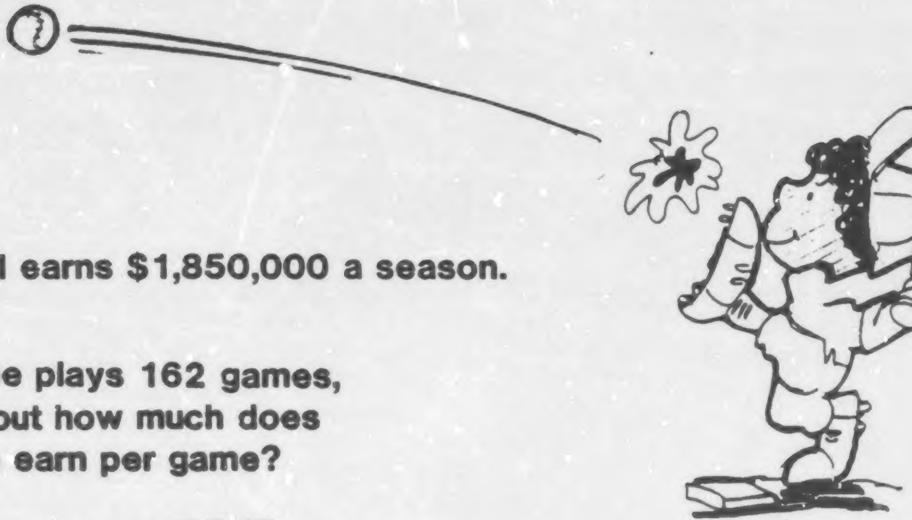
10 100 1000

$56\overline{)1{,}705{,}238}$

100 1000 10000 100000

$365\overline{)587{,}623}$

10 100 1000 10000



Billy B. Ball earns $1,850,000 a season.

If he plays 162 games, about how much does he earn per game?

$162\overline{)1850000}$

Estimate: ______

TRY THESE:

$52\overline{)1850000}$

$12\overline{)1850000}$

Make up a problem about Billy B. Ball that fits these problems.

Name: ____________________

Decide how many digits are in each quotient.

1. $4\overline{)\overset{-\,-\,-}{735}}$ → 3 digits

2. $6\overline{)254}$ → ___ digits

3. $5\overline{)2854}$ → ___ digits

4. $5\overline{)8479}$ → ___ digits

5. $9\overline{)45372}$ → ___ digits

6. $3\overline{)751}$ → ___ digits

7. $4\overline{)53217}$ → ___ digits

8. $7\overline{)43516}$ → ___ digits

9. $8\overline{)9251}$ → ___ digits

10. $7\overline{)29453}$ → ___ digits

Record the lead digit estimate for these quotients.

11. $4\overline{)534}$ with 100 written above: $\overset{100}{534}$

12. $7\overline{)1\overset{-\,-\,-\,-}{5235}}$

13. $6\overline{)1\overset{-\,-\,-}{805}}$

14. $9\overline{)4\overset{-\,-\,-\,-}{6721}}$

15. $5\overline{)2\overset{-\,-\,-}{147}}$

16. $8\overline{)\overset{-\,-\,-}{913}}$

17. $6\overline{)3\overset{-\,-\,-\,-}{1452}}$

18. $9\overline{)3\overset{-\,-\,-\,-}{5476}}$

19. $4\overline{)\overset{-\,-\,-\,-}{5174}}$

20. $8\overline{)7\overset{-\,-\,-\,-\,-}{01546}}$

Decide how many digits will be contained in each quotient then write the lead digit quotient.

21. $\begin{array}{r} 400 \\ 6\overline{)2546} \end{array}$

22. $7\overline{)8321}$

23. $5\overline{)61745}$

24. $9\overline{)4975}$

25. $8\overline{)5584}$

26. $6\overline{)3751}$

27. $7\overline{)43527}$

28. $8\overline{)951076}$

Circle the best estimate for these.

29. $6\overline{)2559}$ 40 400 4000

30. $8\overline{)57056}$ 7 70 700 7000

31. $4\overline{)9012}$ 2 20 200 2000

32. $5\overline{)61243}$ 10 100 1000 10000

NSF ESTIMATION PROJECT
GRADE 8 - LESSON 8

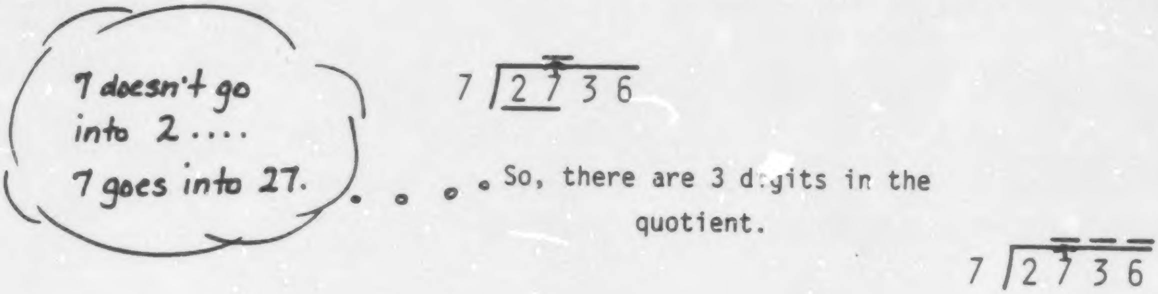
OBJECTIVE : To introduce the compatible numbers strategy for division with 1- and 2-digit divisors.

TEACHER BACKGROUND

Division is typically a difficult operation for students, especially as the size of the numbers become larger. One of the important parts of estimating a quotient is identifying its place value.

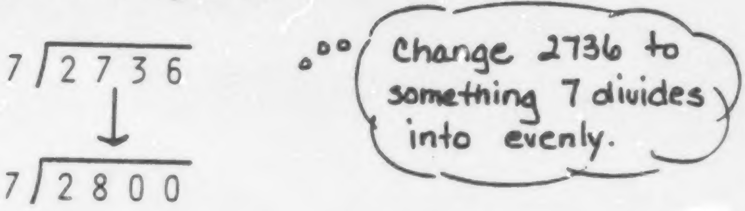
The 3 main steps of the COMPATIBLE NUMBERS strategy for division presented in this lesson are:

1. Identifying the correct place value. This is a very important step which was established in the previous lesson. It will help students avoid place value errors in estimating quotients, which are very common. Here is a review of the thinking process.



So, there are 3 digits in the quotient.

2. Changing the dividend to a number which is compatible with the divisor. The term compatible numbers refers to a set of numbers which are easily computed and always depends on the operation being done.



3. Finally, carry out the division.

$7\overline{)2800}$ is $\underline{\underline{400}}$

In this lesson, students are introduced to this procedure first with one-digit divisors. Next, they apply the procedure to two-digit divisors by rounding the divisor.

Finally, a series of problems are presented where students are free to change both numerator and denominator to form a compatible pair.

TEACHING THE LESSON

PART I. Use TR #1 to help students develop the notion of COMPATIBLE NUMBERS. Show the examples and non-examples one at a time until a student can verbalize what COMPATIBLE NUMBERS are.

Ask students if the same numbers (say 64 and 8) are always compatible? You might ask why 64 and 8 where changed to 60 and 8 in the second example. In fact the second and third examples remind us that whether or not two numbers are compatible depends on the operation involved.

PART II. Use TR #2 to develop the steps of the COMPATIBLE NUMBERS strategy for division.

1. Emphasize thinking about the first steps of the traditional long division algorithm --- where would the first digit be placed? Above which digit of the dividend? Once this is decided, you know how many digits the quotient will contain.
2. Change the dividend so that it is close to the original number but also a multiple of the divisor. In our example, if the dividend were changed to 3200 it would be evenly divisible by 4. The dividend might also be changed to 3600, although this would yield more error. At this point encourage any reasonably close compatible number.
3. Use the information obtained from the first 2 steps to make an estimate.

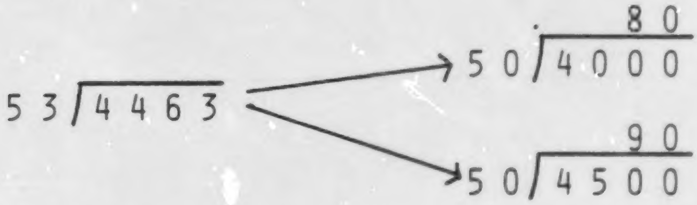
These steps produce a reasonable estimate. However some students may want to make some adjustments. If so, encourage them to adjust the estimate by considering whether the actual quotient is more or less than the initial estimate. In our example, 3342 was rounded down to 3200 so our estimate is an under-estimate. The refined estimate is "over 800." Simply identifying it as over or under takes advantage of information that is easy to obtain and carries the estimation process one step further.

Ask students to do the "TRY THESE." Discussion of their estimates should include their choice of compatible numbers. If adjustments were made, encourage descriptions of the processes used.

PART III. Use TR #3 to illustrate the COMPATIBLE NUMBERS process for a problem with a 2-digit divisor.

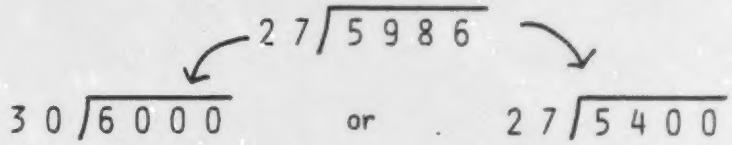
The divisor is first rounded to the nearest ten then the process is carried out just as with 1-digit divisors.

Ask students to do the "TRY THESE." Discussion of their estimates will likely produce different pairs of compatible numbers. For example,



Please encourage the sharing of different compatible numbers and be sure everyone realizes that both estimates (80 and 90) are reasonable.

Some students with a global view may use "other" pairs of compatibles. For example,



Either pair produces a reasonable estimate. It also reminds us that several "different pairs" of compatile numbers may exist for the same problem and provides an entry to the next transparency.

PART IV. In many instances the divisor can be left unrounded. In other cases both dividend and/or divisor can be changed to form a compatible pair. This is a more sophisticated process but one which is an exciting challenge to many students.

TR #4 illustrates several examples of how this process might work. Let your students study these examples and use the process on the "TRY THESE." Encourage students to "think of pairs of compatible numbers" until at least three different pairs have been identified. It should be noticed that "different pairs" will produce "different estimates." This is an important by product of using compatibles and should be recognized by students.

ANSWER KEY

1. 3
2. 4
3. 3
4. 2
5. 2
6. 3
7. 3
8. 4
9. 3

Answers may vary for numbers 10-21

10. $7\overline{)3500}$ or $7\overline{)4200}$
11. $5\overline{)6000}$
12. $8\overline{)4000}$
13. $50\overline{)5000}$
14. $50\overline{)3500}$
15. $30\overline{)12{,}000}$
16. $\frac{144}{12}$ 12
17. $70\overline{)6300}$ 90
18. $50\overline{)450}$ 9
19. $90\overline{)720}$ 8
20. $15\overline{)450}$ 30
21. $25\overline{)750}$ 30
22. $30\overline{)6000}$
    $34\overline{)6800}$
    $35\overline{)7000}$
    $40\overline{)8000}$
23. $25\overline{)7500}$
    $30\overline{)7500}$
24. $14\overline{)2800}$
    $15\overline{)3000}$

25. $40\overline{)3200}$ 80

26. $19\overline{)3800}$ 200

27. $30\overline{)1200}$ 40

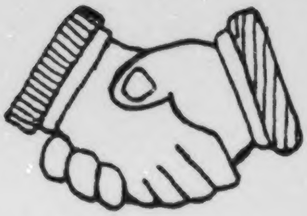
28. about $200

29. about $70

30. about $20

31. about $30

What Are
COMPATIBLE NUMBERS?



These are not compatible numbers.		These are compatible numbers.
$38 + 67 + 49 + 56$	$\longrightarrow$	$35 + 65 + 50 + 50$
64×8	$\longrightarrow$	60×8
$\frac{60}{8}$	$\longrightarrow$	$\frac{64}{8}$
$4\overline{)2637}$	$\longrightarrow$	$4\overline{)2800}$
$19\overline{)3947}$	$\longrightarrow$	$19\overline{)3800}$
$\frac{1}{4} \times 17 \times 968$	$\longrightarrow$	$(\frac{1}{4} \times 16) \times 1000$

Compatible numbers are sets of numbers which are easily computed.

$7\overline{)2249} \longrightarrow\ ?$

Make a
compatible set.

There are 3342 new books
to shelve in the 4 bookcases.

About how many books in each bookcase?

1 Identify the place value of the quotient.

4 doesn't go into 3...
it goes into 33.

$$\begin{array}{r} \overset{-\,-\,-}{\uparrow}\quad \\ 4\overline{)\underbrace{33}42} \end{array}$$

So, there are
3 digits in the answer.

2 Change to compatible numbers.

3342 is
close to 3200.

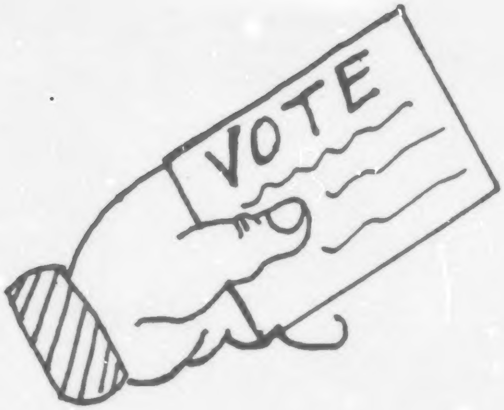
$$\begin{array}{r} -\,-\,- \\ 4\overline{)3342} \end{array}$$

$4\overline{)3200}$

Estimate: **About:** **800**

TRY THESE:

$7\overline{)6148}$ $5\overline{)3789}$ $4\overline{)21,426}$



We need to deliver campaign flyers to the 3684 households of our town. If we have 42 volunteers, how many households should be assigned to each person?

$$42\overline{)3684}$$

① **Round the divisor.** $40\overline{)3684}$

② **Identify the number of digits in the quotient.**

40 goes into 368

$$\begin{array}{r} \_\,\_ \\ 40\overline{)\underbrace{368}4} \end{array}$$

2 digit answer

③ **Change to a compatible number.**

$$\begin{array}{r} \_\,\_ \\ 40\overline{)3600} \end{array}$$

④ **Divide.**

About 90 households for each volunteer

TRY THESE:

$53\overline{)4463}$

$27\overline{)5986}$

$83\overline{)57{,}429}$

LOOK FOR COMPATIBLE PAIRS

Example 1.

$48\overline{)4936} \dashrightarrow 48\overline{)4800}$

I'll leave 48 alone and change 4936 to 4800.

Example 2.

$42\overline{)87943} \dashrightarrow 44\overline{)88000}$

I'll change 42 to 44
and
87,943 to 88,000.

Example 3.

$23\overline{)7146}$

$23\overline{)6900}$ or $25\overline{)7500}$ or $20\overline{)7000}$

TRY THESE:

$19\overline{)7746}$ $34\overline{)6981}$ $27\overline{)5563}$

Name ______________________________

Decide how many digits are in each quotient.

1. 7 /3842

______ digits

2. 5 /6154

______ digits

3. 8 /4376

______ digits

4. 53 /4934

______ digits

5. 48 /3746

______ digits

6. 28 /13,762

______ digits

7. 23 /4973

______ digits

8. 31 /86,432

______ digits

9. 59 /47,643

______ digits

Change each dividend to a compatible number.

10. 7 /3842
7/__________

11. 5 /6154
5 /__________

12. 8 /4376
8 /__________

13. 53 /4934
50 /__________

14. 48 /3746
50 /__________

15. 28 /13,762
30 /__________

Change each division to a compatible number set and divide.

	Change to	Estimated Quotient
16. $\frac{141}{12}$	________	________
17. 69 /6289	________	________
18. 49 /437	________	________
19. 92 /727	________	________
20. 15 /463	________	________
21. 26 /762	________	________

Each problem below has been changed into several forms--some which form compatible sets and some which do not. Circle all sets of compatible numbers for each problem.

22. $34\overline{)7342}$ $30\overline{)6000}$ $30\overline{)7000}$ $30\overline{)7300}$

$34\overline{)6800}$ $35\overline{)7000}$ $40\overline{)8000}$

23. $27\overline{)7746}$ $30\overline{)7700}$ $30\overline{)8000}$ $30\overline{)7500}$

$25\overline{)7700}$ $25\overline{)7500}$

24. $14\overline{)3196}$ $14\overline{)2800}$ $14\overline{)3200}$ $14\overline{)3100}$

$15\overline{)3000}$

Change one or both of the numbers to form a compatible set. Then use the set to divide.

	Change to	Estimate
25. $36\overline{)3152}$	________	________
26. $19\overline{)3764}$	________	________
27. $28\overline{)1278}$	________	________

In the chart are today's credit sales. The purchase price and the number of months the buyer has to pay off the purchase are listed. Estimate the monthly payment for each.

ITEM	TOTAL COST	NUMBER OF MONTHS	ESTIMATED COST PER MONTH
28. Microcomputer	$3798	18	________
29. Video Recorder	$1436	18	________
30. T.V.	$ 467	24	________
31. Stereo	$ 894	30	________

NSF ESTIMATION PROJECT
GRADE 8 - LESSON 9

OBJECTIVES: To review basic concepts involving fractions and decimals less than 1.

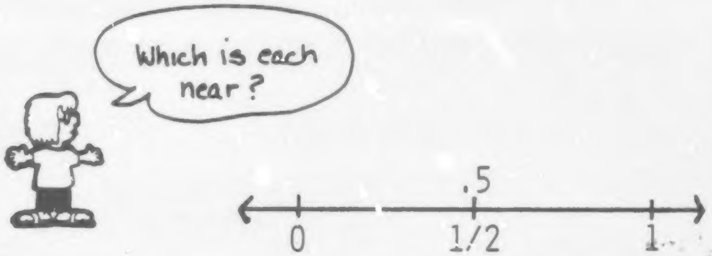
To identify and operate on fractions and decimals near 0, 1/2, and 1.

TEACHER BACKGROUND

According to national assessments, students ability to estimate using fractions and decimals less than 1 is very weak. Some believe this is because of a poor conceptual background concerning rational numbers. In this lesson, attention will be focused on using 0, 1/2, and 1 as reference points when dealing with rational numbers between 0 and 1. Specifically, students will be asked to study various fractions and decimals and to decide which reference number each is closest to. For example,

$\frac{7}{8}$ .4706

$\frac{12}{13}$ .0098496



Several ideas will be emphasized in this lesson.

FRACTIONS

1. Relative size of the fraction compared to the reference values 0, 1/2, and 1. For example,

A fraction is close to . . .

- 0 When the numerator is very small compared to the denominator.
- ½ When the numerator is about ½ the size of the denominator.
- 1 When the numerator is very close in size to the denominator.

2. Is the fraction more or less than it's reference value? For example,

   7/15 and 3/8 are each less than ½ because their numerator is slightly less than half the denominator.

   7/8 is just less than 1 because the numerator is just less than the denominator.

3. Selected Operations involving fractions near 0, 1/2, 1. For example,

   7/8 x 12/13 ∘∘∘ Each is about 1, so the product is a little less than 1.

   4/5 + 3/8 ∘∘∘ That's about $1 + \frac{1}{2}$ or $1\frac{1}{2}$.

DECIMALS

1. Relative size of the decimal compared to reference values 0, .5, and 1. For example,

   .487 is about a half.

   .0049 is about 0.

   .927 is almost 1.

2. Significance of the digits. Emphasis will be focused on the digits immediately following the decimal point. For example,

   Although .004983 contains many digits, it is near 0 because of the digits immediately following the decimal point.

TEACHING THE LESSON

PART I. Review with students the idea of deciding which fractions are near 0, 1/2, and 1 as presented on the top of TR #1. Emphasize comparing numerators and denominators. Ask students to classify each of the fractions given on the transparency.

Focus attention more closely now on specific fractions. Once students have identified 4/9 as near a half - discuss whether it is more or less than 1/2. This could be described in at least 2 ways.

4/9 is less than 1/2 because 4 is less than half of 9.

or

The numerator would have to be 4 1/2 to make the fraction 1/2. Because its less, the fraction is less than 1/2.

TR #2 provides an opportunity to discuss and review these ideas. Some students in your class will find these ideas very easy while others will need more help.

TR #3 provides a discussion of how these ideas can be extended to estimating sums of fractions.

PART II. A parallel treatment of decimals between 0 and 1 is presented on TR #4. However, the emphasis is not on comparing numerators and denominators but focuses on the digit(s) immediately following the decimal point. Remind students that digits past the hundredths place serve more to produce accuracy than anything else. Although .034 is slightly larger than .03 it is a very minor difference in most circumstances. Mentally truncating the digits often lessens confusion over the size of such decimals.

The reference points for these decimals are again 0, .5 (one-half) and 1.

ANSWER KEY

1. $\frac{1}{4}, \frac{4}{11}$

2. $\frac{4}{7}, \frac{5}{9}, \frac{11}{21}$

3. a) $\frac{1}{9}, \frac{1}{100}, \frac{2}{17}$
   b) $\frac{50}{101}, \frac{4}{7}, \frac{9}{19}$
   c) $\frac{85}{86}, \frac{4}{5}, \frac{19}{21}$

4. $\frac{10 \text{ or } 11}{21}$ $\frac{3 \text{ or } 4}{7}$ $\frac{4 \text{ or } 5}{9}$ $\frac{50 \text{ or } 51}{101}$ $\frac{7 \text{ or } 8}{15}$

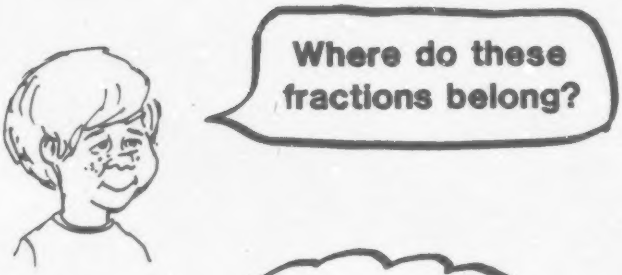
5. Answers may vary:

$\frac{5}{9}$, $\frac{8}{15}$, $\frac{32}{63}$, $\frac{21}{41}$, $\frac{7}{13}$

6. .49, .253, .057
7. .72, .501, .87
8. a) .05, .10111, .1
   b) .516, .459, .50505
   c) .947, .999, .9
9. less than 2
10. less than 1
11. less than 2
12. less than 1
13. less than 2
14. less than ½
15. more than 1
16. more than 1

A fraction is close to....

0	When the numerator is very small compared to the denominator.
$\frac{1}{2}$	When the numerator is about half the size of the denominator.
1	When the numerator is very close in size to the denominator.


Where do these fractions belong?

$\frac{4}{5}$	$\frac{1}{14}$	$\frac{7}{8}$
$\frac{6}{14}$	$\frac{1}{15}$	$\frac{2}{47}$
$\frac{5}{9}$	$\frac{6}{7}$	$\frac{4}{9}$
$\frac{3}{100}$	$\frac{5}{12}$	$\frac{10}{13}$

close to 0

close to $\frac{1}{2}$

close to 1

Finish these fractions so that they are close to but greater than $\frac{1}{2}$.

$\frac{\quad}{8}$ $\frac{\quad}{11}$ $\frac{\quad}{13}$ $\frac{\quad}{21}$ $\frac{9}{\quad}$ $\frac{3}{\quad}$ $\frac{6}{\quad}$

Finish these fractions so that they are close to but less than 1.

$\frac{\quad}{15}$ $\frac{\quad}{9}$ $\frac{\quad}{16}$ $\frac{7}{\quad}$ $\frac{11}{\quad}$ $\frac{9}{\quad}$ $\frac{12}{\quad}$

Fractions Near $\frac{1}{2}$

$\frac{4}{9}$ $\frac{5}{9}$ $\frac{8}{14}$ $\frac{16}{32}$

$\frac{10}{21}$ $\frac{8}{16}$ $\frac{5}{11}$ $\frac{7}{15}$ $\frac{19}{40}$

Which fractions are just less than $\frac{1}{2}$?

Which fractions are just more than $\frac{1}{2}$?

Which fractions are exactly $\frac{1}{2}$?

TRY THESE:

Use only these numbers: 1, 2, 3, 4, 5.

Make a fraction that is:

$\frac{\square}{\square}$

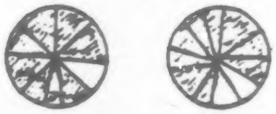
- less than $\frac{1}{2}$
- exactly $\frac{1}{2}$
- more than $\frac{1}{2}$
- close to 0
- close to 1
- more than 1
- less than 1

These fractions are all just less than one.

$\frac{4}{5}$ $\frac{5}{6}$ $\frac{8}{9}$ $\frac{7}{8}$ $\frac{9}{11}$ $\frac{11}{12}$

Suppose two of them are added . . .

$\frac{8}{9} + \frac{9}{11}$



Together, it's less than 2 pies.

The sum must be . . .

less than 2 **exactly 2** **more than 2**

Suppose three of these fractions are added . . . the sum must be ______________.

Think!

Suppose two fractions, each less than $\frac{1}{2}$, are added.

The sum must be ____________. Why?

Suppose three fractions, each greater than $\frac{1}{2}$, are added.

The sum must be ____________. Why?

TRY THESE:

1. $\frac{7}{8} + \frac{4}{5}$ **less than** ___ **more than** ___

2. $\frac{1}{3} + \frac{7}{15}$ **less than** ___ **more than** ___

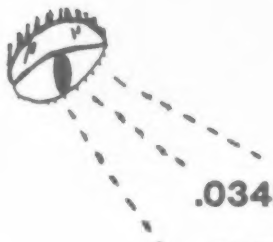
3. $\frac{5}{9} + \frac{7}{13} + \frac{2}{3}$ **less than** ___ **more than** ___

4. $\frac{4}{5} + \frac{11}{12} + \frac{8}{9}$ **less than** ___ **more than** ___

To Estimate The Size Of A Decimal . . . .

Look at the digits just after the decimal point.

Ignore the rest.



.0345601 . . . . . . is near 0

.47803 . . . . is near .5 (one-half)

.91 . . . . is near 1

Let's practice. . . . Identify these as near 0, $\frac{1}{2}$, or 1.

.009879 .9003 .90000001

.89670001 .01049 .49

.50023 .4701 .01

Estimate. Use what you know about each decimal.

.9046 + .4900031

.003029 + .97

.960 - .502891

.510 + .480009 + .89901

.0003998 + .925 + .487001

.50781 + .492 - .000034 - .5023009

Name ____________________

1. Circle the fractions less than $\frac{1}{2}$.

$\frac{1}{4}$ $\frac{2}{3}$ $\frac{5}{9}$

$\frac{4}{11}$ $\frac{5}{8}$ $\frac{11}{21}$

2. Circle the fractions greater than $\frac{1}{2}$.

$\frac{2}{7}$ $\frac{4}{7}$ $\frac{5}{9}$

$\frac{3}{8}$ $\frac{8}{17}$ $\frac{11}{21}$

3.

$\frac{85}{86}$ $\frac{1}{9}$ $\frac{1}{100}$ $\frac{4}{5}$ $\frac{50}{101}$ $\frac{19}{21}$ $\frac{4}{7}$ $\frac{2}{17}$ $\frac{9}{19}$

List the three fractions closest to 0: ____ ____ ____

closest to $\frac{1}{2}$: ____ ____ ____

closest to 1: ____ ____ ____

4. Fill in the numerator to make these fractions near $\frac{1}{2}$:

$\frac{\_\_}{21}$ $\frac{\_\_}{7}$

$\frac{\_\_}{9}$ $\frac{\_\_}{101}$

$\frac{\_\_}{15}$

5. Fill in the denominator to make these fractions near $\frac{1}{2}$ but greater than $\frac{1}{2}$.

$\frac{5}{\_\_}$ $\frac{8}{\_\_}$

$\frac{32}{\_\_}$ $\frac{21}{\_\_}$

$\frac{7}{\_\_}$

6. Circle the decimals less than one-half.

.49 .253

.6 .514

.7 .057

7. Circle the decimals greater than one-half.

.72 .09

.007 .4999

.501 .87

8.

.05 .947 .999 .50505

.516 .459 .10111 .1 .9

List the three decimals closest to 0: ______ ______ ______

closest to $\frac{1}{2}$: ______ ______ ______

closest to 1: ______ ______ ______

Use what you know about fractions and decimals to estimate the best answer.

9. $\frac{7}{8} + \frac{9}{10}$ is
more than 2
less than 2

10. $\frac{4}{10} + \frac{3}{7}$ is
more than 1
less than 1

11. $\frac{5}{7} + \frac{8}{9}$ is
more than 2
less than 2

12. .492 + .4857 is
more than 1
less than 1

13. .9 + .9571 is
more than 2
less than 2

14. $\frac{1}{8} + \frac{1}{15}$ is
more than $\frac{1}{2}$
less than $\frac{1}{2}$

15. $\frac{6}{11} + \frac{7}{13}$ is
less than 1
more than 1
more than 2

16. .517 + $\frac{8}{15}$ is
less than 1
more than 1
more than 2

OBJECTIVE : To estimate problems of the type 1/4 of N.

To extend 1/4 of N to produce an estimate for 3/4 of N.

To develop the idea of converting uncommon fractions within an estimation situation to approximate common fractions.

TEACHER BACKGROUND

Many everyday situations use fractions and most of these involve common fractions -- 1/10, 1/4, 1/3, 1/2, 2/3, 3/4, 9/10. Others deal with fractions of a more precise nature which often forces them to be a bit messier -- 7/16, 3/13, 29/38.

This lesson will focus on how estimation with both common and messy fractions can be accomplished. Basically, the strategy will be to change any "messy" fraction to an approximately equivalent common fraction. Then compute with this common fraction and adjust the final estimate to reflect the change.

This strategy provides further evidence of the power and diversity of compatible numbers. As with other uses of the strategy, the common fractions selected depend upon the operation as well as the numbers involved. Effective use of this strategy requires that students have a good understanding of fractions. They also need practice in changing a "messy" fraction to a near common fraction and practice computing with common fractions. This lesson will provide such practice.

TEACHING THE LESSON

PART I. In estimating with fractions, it is important for students to be able to identify and use the more common fractional amounts (for example, 1/4, 1/3, 1/2, 2/3, 3/4, or 9/10). In this lesson, we refer to these fractions as "nice" fractions because they are easy to mentally compute with. List these fractions on the board and discuss why they are called "nice" fractions.

- "Nice" fractions are those commonly used in everyday situations.
- "Nice" fractions are easier to mentally compute with than other fractions.

Use TR #1 to review how "nice" fraction computation can be done mentally.

Multiplying by unit fractions (fractions whose numerator is 1) is simply a matter of dividing. For example:

$\frac{1}{4}$ of 280 ... That's the same as $280 \div 4$ or 70.

Practice this idea using the top portion of TR #1. For problems which involve a multiple of a unit fraction, this same process can be extended ...

$\frac{3}{4}$ of 280 ... That's 3 times $\frac{1}{4}$ of 280 or 210.

PART II. For estimating problems such as 1/3 of 290, we need to change the 290 so that it is compatible with 1/3. 1/3 of 290 is about 1/3 of 300 or 100-. Let students discuss this idea then practice it using the problems on TR #2. You might present these problems orally one at a time or allow students several minutes to do the set independently then discuss the answers.

PART III. Suppose we need to make an estimate involving fractions which are not so "nice"? For example, how will we estimate 23/49 of 70? This is a fairly easy estimation problem IF we can decide what 23/49 approximates.

Review the following ideas discussed in Lesson 9.

A fraction is close to:

- 0 when the numerator is small compared to the denominator.
- $\frac{1}{2}$ when the numerator is about $\frac{1}{2}$ the size of the denominator.
- 1 when the numerator is very close to the denominator.

Use TR #3 to begin discussion of how "messy" fractions can be estimated. What "nice" fraction is 23/49 near? It is a little less than 1/2 because the numerator is just slightly less than half the denominator. So, when estimating, 1/2 can be used to replace 23/49.

$$\frac{23}{49} \text{ of } 50 \approx \frac{1}{2} \text{ of } 50 \text{ or } \underline{\underline{25}}$$

Use the middle of TR #3 to help students practice finding "nice" fractions near each of the fractions listed. Their first step should be to identify whether the fraction is less than or greater than 1/2. Next, they can change the numerator and/or denominator slightly to search for a "nice" number approximation.

The bottom of TR #3 provides opportunity for students to apply these "nice" fraction approximations in estimating. Encourage students to first decide on the "nice" fraction to be used, next find a compatible number for it, then finally, carry out the mental computation for their estimate.

ANSWER KEY

1. 4
2. 6
3. 8
4. 16
5. 16
6. 32
7. 60
8. 180
9. 510
10. 1020
11. 12
12. 630

Answers may vary for exercises 13-20.

13. $50, $25
14. $120, $40
15. $120, $80
16. $80, $20
17. $80, $60
18. $60, $10

19. $50, $10
20. $3500, $1400
21. 2/3
22. 1/4
23. 1/10
24. 1/2
25. 1/2
26. 1/3
27. 9/10
28. 3/4
29. 2/3
30. about $340
31. about 400
32. about 700
33. about 120
34. about 120
35. about 85
36. about 80,000
37. $25, $50
38. a) $30,000-$31,000
    b) $2500-$2600
    c) $70,000-$80,000

Mentally Compute

$\frac{1}{2}$ of 90 $\frac{1}{4}$ of 280 $\frac{1}{3}$ of 360

$\frac{1}{5}$ of 800 $\frac{1}{10}$ of 940

$\frac{1}{2}$ of 9600 $\frac{1}{3}$ of 6300 $\frac{1}{4}$ of 8400

Use $\frac{1}{4}$ of 280 to find $\frac{3}{4}$ of 280

That's 3 times $\frac{1}{4}$ of 28

⇓ ⇓

____ so. . . . ____

TRY THESE:

$\frac{1}{3}$ of 660 $\frac{2}{3}$ of 660

$\frac{1}{10}$ of 250 $\frac{3}{10}$ of 250 $\frac{7}{10}$ of 250

$\frac{1}{5}$ of 300 $\frac{2}{5}$ of 300 $\frac{4}{5}$ of 300

Change to a compatible set, then compute. . . .

$\frac{1}{6}$ of 117

- Search for a compatible set.

  $\frac{1}{6}$ of 120

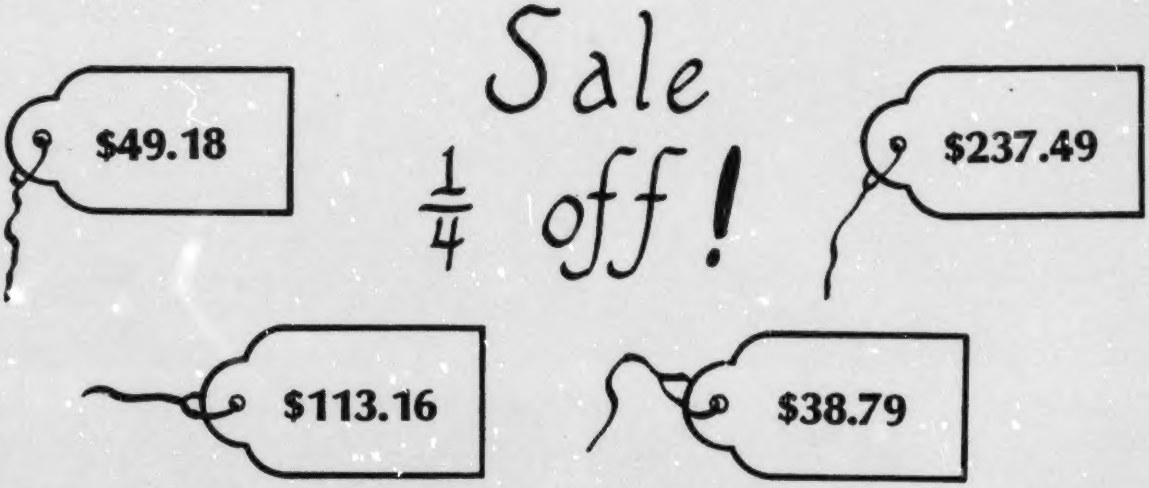
- Compute.

  20

TRY THESE:

$\frac{1}{4}$ of 317	$\frac{3}{4}$ of 317
$\frac{1}{3}$ of 281	$\frac{2}{3}$ of 281
$\frac{1}{5}$ of 5417	$\frac{3}{5}$ of 5417
$\frac{1}{8}$ of 7246	$\frac{3}{8}$ of 7246

TRY THESE: Estimate the discount.


Sale
$\frac{1}{4}$ off!
$49.18
$237.49
$113.16
$38.79

Estimate:

$\frac{23}{49}$ of 720

Clean the problem up. Think. . .

- What common fraction is $\frac{23}{49}$ near?

It's about $\frac{1}{2}$!

- Change it.

$\frac{1}{2}$ of 720

- Compute.

That's 360!

Let's practice. Name a common fraction near each of these.

$\frac{4}{9}$ $\frac{5}{14}$ $\frac{17}{18}$ $\frac{14}{45}$ $\frac{7}{29}$ $\frac{16}{30}$

TRY THESE:

Estimate. . .

$\frac{4}{9}$ of 679

$\frac{14}{45}$ of 3219

$\frac{5}{14}$ of 889

$\frac{7}{29}$ of 878

$\frac{17}{18}$ of 4204

$\frac{16}{30}$ of 840

Name ____________________

Mentally compute:

1. $\frac{1}{6}$ of 24 = ________
2. $\frac{1}{3}$ of 18 = ________
3. $\frac{1}{5}$ of 40 = ________
4. $\frac{2}{5}$ of 40 = ________
5. $\frac{1}{3}$ of 48 = ________
6. $\frac{2}{3}$ of 48 = ________
7. $\frac{1}{4}$ of 240 = ________
8. $\frac{3}{4}$ of 240 = ________
9. $\frac{1}{3}$ of 1530 = ________
10. $\frac{2}{3}$ of 1530 = ________
11. $\frac{3}{4}$ of 16 = ________
12. $\frac{1}{2}$ of 1260 = ________

Rewrite using a compatible number. Then use it to make an estimate.

13. $\frac{1}{2}$ of $49.95 → $\frac{1}{2}$ of ________ Estimate: __________
14. $\frac{1}{3}$ of $118.99 → $\frac{1}{3}$ of ________ Estimate: __________
15. $\frac{2}{3}$ of $118.99 → $\frac{2}{3}$ of ________ Estimate: __________
16. $\frac{1}{4}$ of $78.95 → $\frac{1}{4}$ of ________ Estimate: __________
17. $\frac{3}{4}$ of $78.95 → $\frac{3}{4}$ of ________ Estimate: __________
18. $\frac{1}{6}$ of $61.36 → $\frac{1}{6}$ of ________ Estimate: __________
19. $\frac{1}{5}$ of $48.88 → $\frac{1}{5}$ of ________ Estimate: __________
20. $\frac{2}{5}$ of $3488 → $\frac{2}{5}$ of ________ Estimate: __________

Find a "nice" fraction from this list that is near each "messy" fraction.

$\frac{1}{10}$ $\frac{1}{4}$ $\frac{1}{3}$ $\frac{1}{2}$ $\frac{2}{3}$ $\frac{3}{4}$ $\frac{9}{10}$

21. $\frac{20}{31}$ ________

22. $\frac{5}{19}$ ________

23. $\frac{9}{91}$ ________

24. $\frac{15}{31}$ ________

25. $\frac{19}{40}$ ________

26. $\frac{9}{28}$ ________

27. $\frac{92}{100}$ ________

28. $\frac{61}{80}$ ________

29. $\frac{40}{59}$ ________

Use "nice" fractions to estimate the result.

30. $\frac{15}{31}$ of $680 ________

31. $\frac{7}{29}$ of 1599 ________

32. $\frac{10}{51}$ of 3495 ________

33. $\frac{8}{25}$ of 360 ________

34. $\frac{40}{59}$ of 180 ________

35. $\frac{10}{99}$ of 850 ________

36. The newspaper reported that unemployment had reached $\frac{1}{4}$ of the working force in Detroit. Estimate the number of unemployed people if Detroit has a working population of 323,400.

37. Estimate the discount and the new price on a tennis racket which sells for $74.95 and is advertised to be 1/3 off.

DISCOUNT: ________ SALE PRICE: ________

38. Property tax is based on the assessed valuation of property. The assessed valuation is 1/3 of the real value. Estimate the assessed valuation of the following properties:

Property	Real Value	Assessed Valuation
House	$92,500	________
Car	$7,600	________
Farm	$237,500	________

NSF ESTIMATION PROJECT
GRADE 8 - LESSON 11

OBJECTIVES: To review and practice the COMPATIBLE NUMBERS strategy for whole number division.

To extend the COMPATIBLE NUMBERS concept to mixed operations.

TEACHER BACKGROUND

This lesson reviews the COMPATIBLE NUMBERS strategy for division and extends it to multi-step problems. COMPATIBLE NUMBERS are those combinations of numbers which permit mental computation. For example, $289 \div 14$ can be thought of as $280 \div 14$ since this is easier to mentally compute than $289 \div 14$.

Students should be made aware that there are frequently different sets of COMPATIBLE NUMBERS which will each provide good estimates. For example, $289 \div 14$ can be thought of as $300 \div 15$ (or about 20). This is especially true when more than 2 numbers are involved or it is a multi-operation problem. An example will help illustrate this idea.

$$\frac{321 \times 9}{17}$$

could be expressed as ---

1. $320 \times \frac{1}{2}$ or 160

2. $\frac{340}{17} \times 9$ or 180

3. $\frac{320 \times 10}{20}$ or 160

Throughout this lesson encourage students to discuss the different possible sets of COMPATIBLE NUMBERS for each problem. They should also begin to develop generalizations about which numbers in the problem are more efficiently changed.

TEACHING THE LESSON :

PART I. Use TR #1 to review the COMPATIBLE NUMBERS strategy for division taught in Lesson #8. Emphasize that the strategy allows you to change the numbers so that they permit mental computation. There are different ways these problems can be reformed so all reasonable combinations should be allowed.

The exercises at the bottom of the transparency provide practice in searching for and using COMPATIBLE NUMBERS. Sharing different solutions to these exercises will strengthen students' skills with COMPATIBLE NUMBERS.

PART II. TR #2 illustrates how the COMPATIBLE NUMBERS strategy can be used when we are estimating multi-step problems with both multiplication and division. Again, students should initially step back and take a global look at the entire problem before searching for a way to take advantage of COMPATIBLE NUMBERS. Without this search, the problem could be unmanageable.

PART III. TR #3 illustrates how fractions are often used with COMPATIBLE NUMBERS. Students should be aware that they have the flexibility to replace some numbers in the computations with COMPATIBLE NUMBERS. Once these choices are made they can mentally compute the estimate using a logical and convenient order of the operations.

Mastery of this work with COMPATIBLE NUMBERS and multi-step problems takes time. Nevertheless understanding this process provides powerful estimation skills that have wide application.

USING THE EXERCISES:

Students will likely need more practice to feel comfortable with the concept presented in this lesson. You may wish to do the student sheets in class, discussing various problems as students work.

ANSWER KEY:

1. 45/15 3
2. 720/80 9
3. $19 \overline{)380}$ 20
4. 3600/40 90
5. $25 \overline{)7500}$ 300
6. about 6
7. 20-¢
8. $2000-, $400+, $100-
9. a) 4 x 600 = 2400
   b) 300 x 8 = 2400

10. a) $20 \times 14 = 280$

b) $600 \times \frac{1}{2} = 300$

c) $\frac{6000}{30} = 200$

11. a) $8 \times 2500 = 20,000$

b) $5000 \times 4 = 20,000$

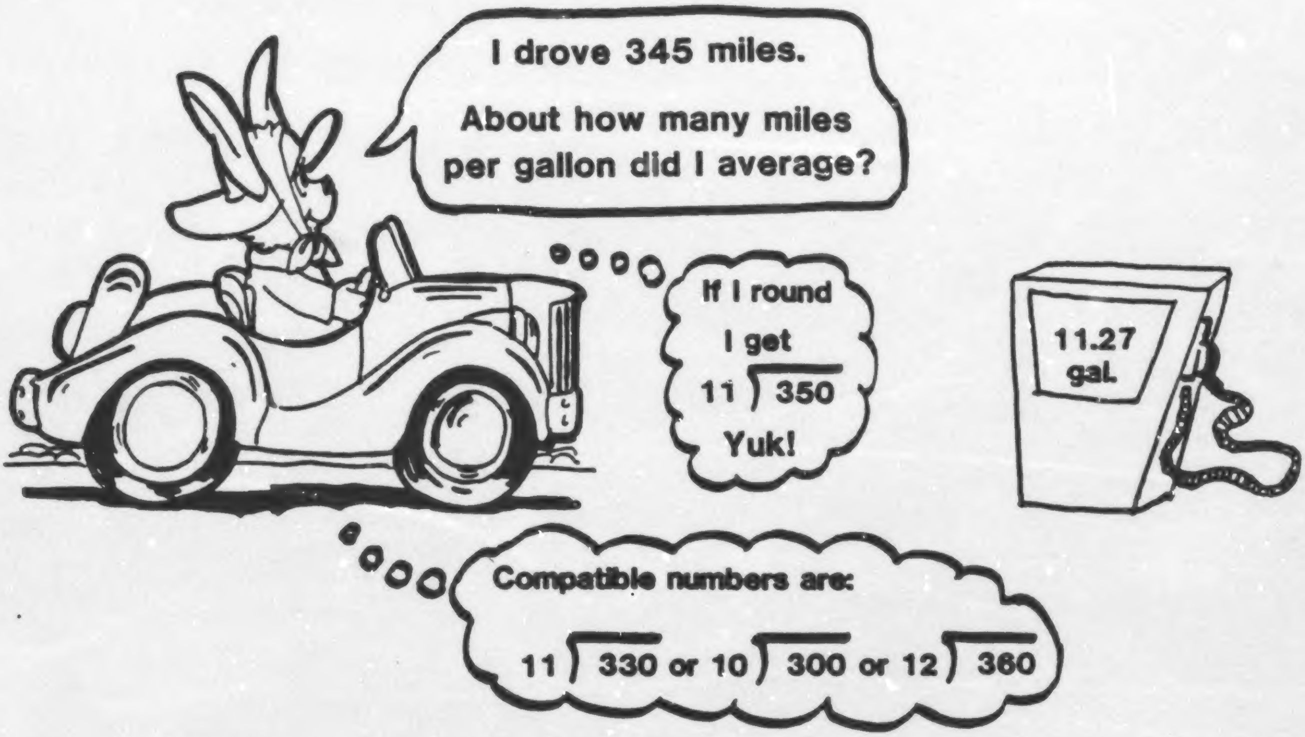
12. $\frac{180}{90} \times 360 = 720$ or $200 \times \frac{360}{90} = 800$ or $190 \times \frac{360}{90} = 760$

13. 3000-3600

14. 400-500

15. 7-8

16. 14-24


I drove 345 miles.
About how many miles per gallon did I average?
If I round I get 11) 350 Yuk!
11.27 gal.
Compatible numbers are:
11) 330 or 10) 300 or 12) 360

What makes these numbers **compatible**?

What is your estimate?

TRY THESE:

Find compatible numbers for each of these, then estimate. Try to do them all in 2 minutes!

1. $\frac{29}{15}$

2. $\frac{713}{92}$

3. $15\overline{)\,448}$

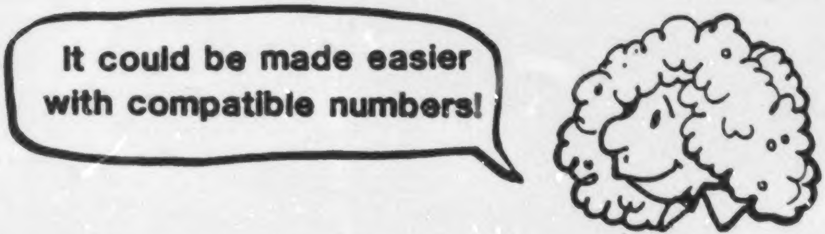
4. $12\overline{)\,148}$

5. $23\overline{)\,7486}$

6. $\frac{2408}{59}$

How can we estimate:

$$\frac{348 \times 17}{9}$$


It could be made easier with compatible numbers!

We could:

$$350 \times 2$$
$$700$$

or

$$\frac{360}{9} \times 20$$
$$40 \times 20$$
$$800$$

or

$$\frac{350 \times 20}{10}$$
$$350 \times 2$$
$$700$$

TRY THESE:

Use a compatible set of numbers!

1. $\dfrac{478 \times 36}{7}$
2. $\dfrac{716 \times 65}{8}$
3. $\dfrac{96 \times 217 \times 58}{19}$
4. $\dfrac{94 \times 198 \times 14}{7 \times 46}$

Let's estimate:

$$\frac{9 \times 17}{35}$$

$$\frac{9}{36} \times 20$$

$$\frac{1}{4} \times 20$$

5

or

$$9 \times \frac{17}{34}$$

$$9 \times \frac{1}{2}$$

$4\frac{1}{2}$

TRY THESE:

1. $\dfrac{6 \times 19}{37}$

2. $\dfrac{15 \times 37 \times 491}{62}$

3. $\dfrac{18 \times 59}{21}$

4. $\dfrac{94 \times 46 \times 14}{7 \times 198}$

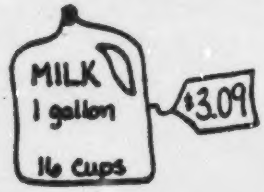
Name ____________________

Find compatible numbers for each problem. Then estimate.

	Change to	Estimate
1. $\frac{46}{15}$	__________	__________
2. $\frac{719}{83}$	__________	__________
3. $19\overline{)389}$	__________	__________
4. $\frac{3509}{39}$	__________	__________
5. $26\overline{)7643}$	__________	__________

6. The fastest barber on record shaved 368 faces in one hour. About how many faces were shaved each minute? __________

7.



At this price, about how much does a cup of milk cost? __________

8. My salary is $22,700 per year. About how much do I make per month? __________ per week? __________ per day? __________

Each problem below has been set up several ways using compatible numbers. Find the estimate using each way.

9. $\frac{293 \times 649}{78}$

(a) $\frac{280}{70} \times 600$ (b) $300 \times \frac{640}{80}$

10. $\frac{613 \times 14}{27}$

(a) $\frac{600}{30} \times 14$ (b) $600 \times \frac{1}{2}$ (c) $\frac{600 \times 10}{30}$

11. $\frac{4956 \times 2389}{612}$

(a) $\frac{4800}{600} \times 2500$ (b) $5000 \times \frac{2400}{600}$

12. Find 2 ways to do this problem.

$$\frac{186 \times 364}{91}$$

Make estimates for these:

13. $\frac{561 \times 412}{73}$

14. $\frac{31 \times 435 \times 19}{59 \times 9}$

15. $\frac{652}{21 \times 4}$

16. $\frac{312 \times 248 \times 28}{39 \times 49 \times 58}$

NSF ESTIMATION PROJECT
GRADE 8 - LESSON 12

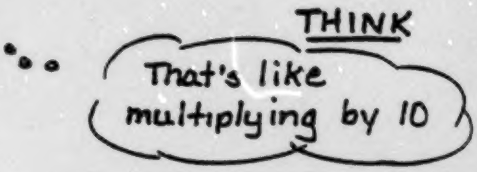
OBJECTIVE: To develop estimation skills when multiplying or dividing by numbers near a power of ten.

TEACHER BACKGROUND

Students are introduced to the "nice" numbers estimation strategy in this lesson. "Nice" numbers, as we are using the term, are those numbers which allow mental computation to be performed quickly and easily. "Nice" numbers considered here are powers of ten (e.g. 1, 10, 100).

The "nice" numbers strategy involves a technique in which students round selected numbers to "nice" numbers (e.g. $1.03 \approx 1$; $9.94 \approx 10$) and then perform the indicated operations. After these operations are performed, a study is made to see if the refined estimate should be a little more or a little less. The following example illustrates the entire process.

45.6×9.98

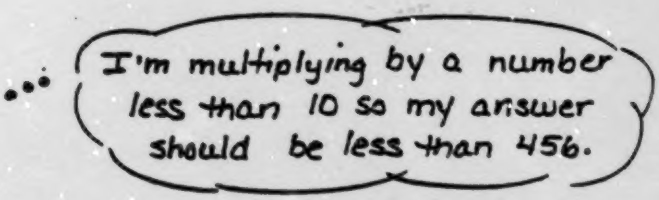


45.6×10

456

less than 456

or 456^-


I'm multiplying by a number less than 10 so my answer should be less than 456.

Prior to teaching this lesson you may want to review multiplication and division by powers of ten.

TEACHING THE LESSON

PART I: Provide examples of "Nice Numbers" using TR #1. "Nice Numbers" are those which make computation quick and easy. Point out that frequently numbers occur which are near these "nice" numbers. Give several examples -- then have students add to the list. Use the bottom of the transparency to have students select the numbers which are near "nice" numbers and state which "nice" numbers they are near.

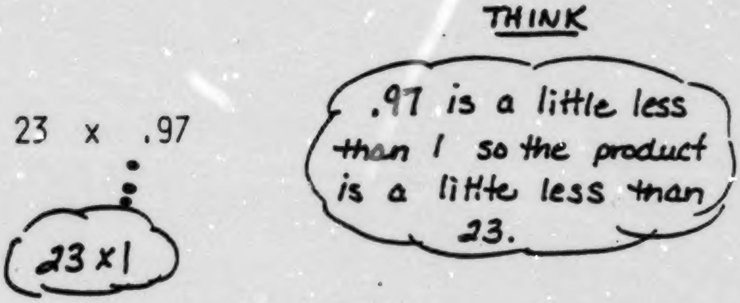
Note: The acceptable range for these numbers will be determined by the context of the problem in real-life situations. However, for our lesson, numbers near 10 will generally be between 9 and 11, while numbers near 100 will generally be between 95 and 105.

PART II: Use the Calculator Worksheet (TR #2) to begin collecting and recording estimates for problems such as 23 x .97.

Ask for estimates of the product from several students. For example,

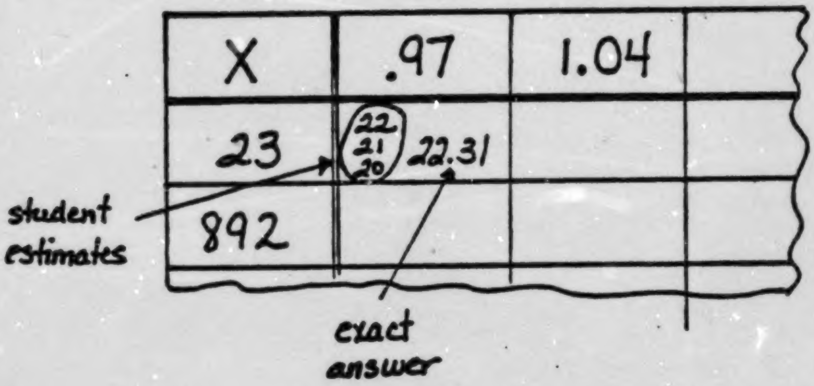
23 x .97	What is .97 near? What will the product be near?

Encourage students to think of .97 as "a little less than one." For example,


THINK
23 x .97
23 x 1
.97 is a little less than 1 so the product is a little less than 23.

ESTIMATE: "A little less than 23, I'll say 22."

These estimates should be recorded, then the exact answer can be found using the calculator.


student estimates
exact answer

X	.97	1.04	
23	22 21 20 22.31		
892			

Continue in this manner estimating 892 x .97. Repeat this process with several other numbers near nice numbers (e.g. 1.04, 9.8, 10.9, 97.2, 103). Pick and choose problems from the chart as you wish. Students will not need to complete the whole chart before going on.

Students should be encouraged to seek a pattern and verbalize this pattern when observed.

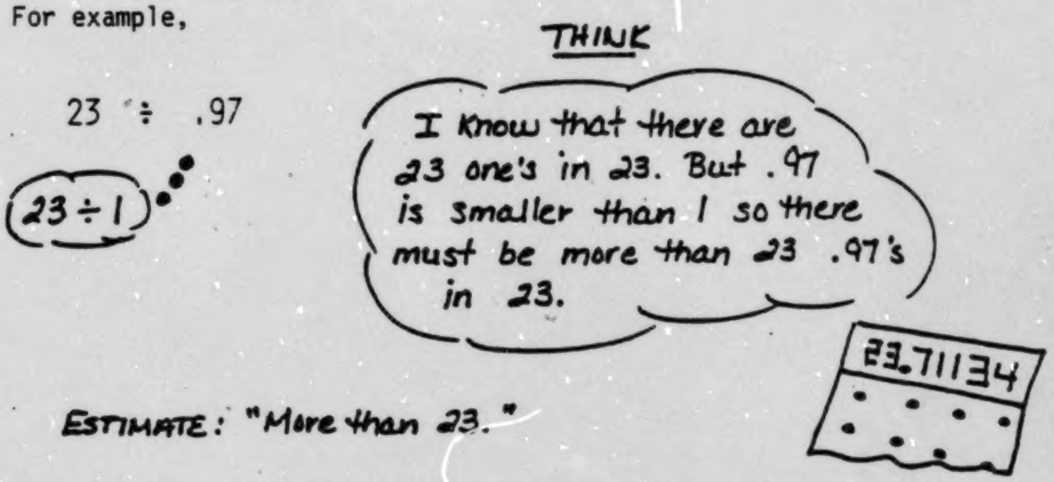
Pattern: Given a number near N, where N = a power of ten, the product of this number and any other number is found by multiplying by N and then adding or subtracting an adjustment (compensation) determined by whether the given number was greater than or less than N.

The compensation (adding or subtracting the adjustment) is a complex process. Students need to realize that the degree of accuracy depends on the situation, but exactness is not the objective. In this lesson, stress determination of whether the exact answer is more than or less than the initial estimate.

PART III. Use the same process for division by near-nice numbers on TR #2. Help the students notice that the pattern is now reversed. When dividing by a number greater than N, the adjustment is subtracted. When dividing by a number less than N, the adjustment is added. This is a more complex concept and will need plenty of discussion.

For example,

23 ÷ .97


THINK
23 ÷ 1
I know that there are 23 one's in 23. But .97 is smaller than 1 so there must be more than 23 .97's in 23.
23.71134

ESTIMATE: "More than 23."

Students will need to see a variety of examples to help them incorporate this idea.

PART IV: Supervised Practice (use TR #3)

Refer to the two examples provided and then have the students try the six problems. It is suggested that students share estimating strategies orally with the class. Specifically, they should be encouraged to verbalize how they move from their rough estimate to their refined estimate.

In many cases the refined estimate could simply be expressed by using the words "more than" or "less than" and written with the symbols + or -. For example,

Problem	Estimate	
	Rough	Refined
430 x 10.2	4300	4300+
430 ÷ 10.2	43	43-

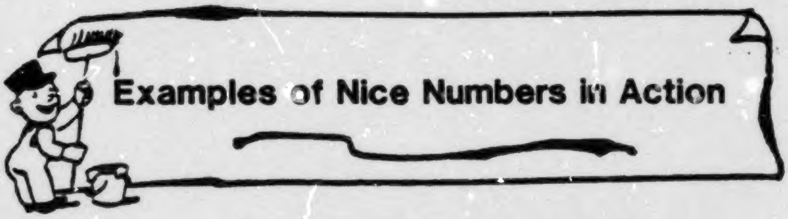
USING THE EXERCISES

Worksheet #1 should be done immediately following the presentation of the lesson. It will give you a quick check on how well the students have picked up on the ideas presented.

ANSWER KEY

1. 4600 4600- or 4500
2. 782 782+ or 785
3. 840 840+ or 845
4. 673 673+ or 680
5. 5420 5420+ or 5430
6. 29.8 29.8+ or 30
7. 425,000 425,000+ or 430,000
8. 86.7 86.7+ or 87
9. 342 342+ or 350
10. 168 168+ or 170
11. 520 520- or 510
12. 8.46 8.46- or 8
13. 12.38 12.38+ or 13
14. 436.7 436.7- or 436

15. 559.55
16. 8654.69
17. 2.999
18. 80.86
19. 41.28
20. 86.57
21. 53990.74
22. 6.999
23. $1.40-$1.60
24. yes
25. less
26. $475-
27. 32.5-
28. 37+
29. 2600+ miles
30. $5.40+
31. $110,000-
32. more



Examples of Nice Numbers in Action

3 x 1 40 ÷ 10 58 x 100

85 ÷ 1 67 x 10 2500 ÷ 100

Why are the underlined numbers called "nice" numbers?

** Numbers Near Nice Numbers **

Near 1

.9

1.1

.97

1.13

1.0143

Near 10

9.6825

11

10.2

9.9

10.67

Near 100

103

104.3

95.6

98.5473

99

**** Find the numbers below which are near nice numbers and state which nice numbers they are near. ****

1.09 23.4 781 3.6 .96

78 9.86 .45 102.9375

97 1.127 17.2 10.4 135

Calculator Worksheet

X	Near 1		Near 10		Near 100	
	.97	1.04	9.8	10.9	97.2	103.
23						
892						

÷	Near 1		Near 10		Near 100	
	.97	1.04	9.8	10.9	97.2	103.
23						
892						

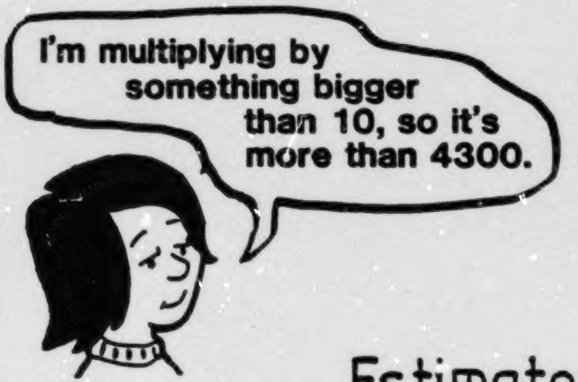
Estimate: 430×10.231

Think

$430 \times 10.231 \approx 430 \times 10$

4300

Rough estimate


I'm multiplying by something bigger than 10, so it's more than 4300.

4300^+

or about 4400

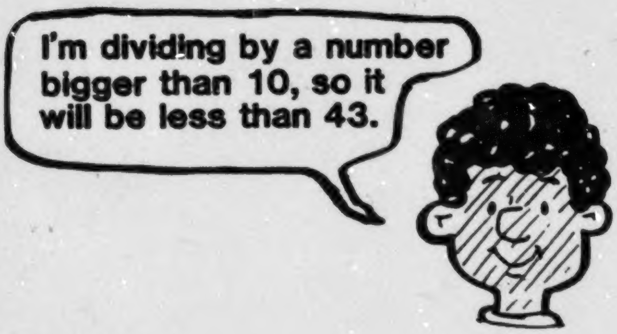
Refined estimate

Estimate: $430 \div 10.231$

$430 \div 10.231 \approx 430 \div 10$

43

Rough estimate


I'm dividing by a number bigger than 10, so it will be less than 43.

43^-

or about 42

Refined estimate

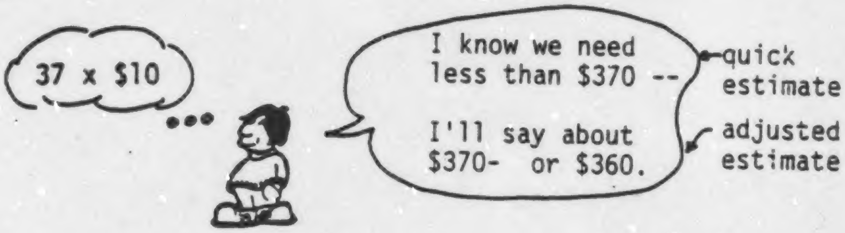
TRY THESE:

1. $485 \times .985$
2. $7800 \div 9.61$
3. 175×10.53
4. 56×97.8
5. $.862 \div 1.03$
6. $35{,}000 \div 104$

8-12-TR3

NAME ______________________________

We need 37 Fun Day tickets
They cost $9.75 each.


37 x $10
I know we need less than $370 --
I'll say about $370- or $360.
quick estimate
adjusted estimate

Make a quick estimate on these and then adjust.

		QUICK	ADJUSTED			QUICK	ADJUSTED
1.	460 x 9.75	______	______	2.	782 x 1.05	______	______
3.	84 x 10.3	______	______	4.	673 x 1.08	______	______
5.	542 x 10.29	______	______	6.	29.8 x 1.1	______	______
7.	425 x 1015	______	______	8.	86.7 ÷ .98	______	______
9.	3420 ÷ 9.78	______	______	10.	16,800 ÷ 99	______	______
11.	52,000 ÷ 102.7	______	______	12.	8460 ÷ 1029	______	______
13.	12,380 ÷ 991	______	______	14.	4367 ÷ 10.04	______	______

Here are the calculator results. One is correct, the other is wrong. Use estimation to help find the correct answer and circle it.

15.	589 x .95	559.55	629.945
16.	89576 ÷ 10.35	8654.69	9271.16
17.	35.75 ÷ 11.92	426.14	2.999
18.	87 ÷ 1.076	80.86	93.61
19.	387 ÷ 9.375	36.28	41.28
20.	85,397 ÷ 986.43	86.57	83.64
21.	53.6 x 1007.29	53,990.74	52,864.50
22.	6.7 ÷ .9573	6.257	6.999

Use estimation to find reasonable answers for these problems.

23. Your grocery store sells hamburger for $1.39 per pound. If you buy a package of hamburger that weighs 1.08 lb., estimate what you will pay. ________________

24. Suppose that 1 liter bottles of cola are on sale for 99¢ a piece and you are going to buy 24 bottles for the church picnic. Will $24.85 be enough money to pay the bill? ________________

25. A racer averages 9.9 miles per hour when running long distances. Will a marathon race about 26 miles long take more or less than 3 hours? ________________

26. The owner of the Sound Shop has 475 tapes in inventory and makes about 95¢ on each tape sold. Estimate the profit if all of the tapes were sold. ________________

27. The Brocks drove 325 miles on 10.2 gallons of gasoline. Estimate their mileage. ________________

28. The Harts drove 370 miles on 9.85 gallons of diesel fuel. Estimate their mileage. ________________

29. Let's suppose that a cross country train is able to maintain an average speed of 107 mph from 8:30 a.m. until 10:30 a.m. the next day. Estimate the distance the train would cover in this amount of time. ________________

30. There are 97 students in the 8th grade class of Jefferson Jr. High. They are planning a trip which will cost a total of $540. Estimate the amount each student whould pay to cover the expenses of the trip. ________________

31. If 9,960 people bought tickets to see REO Speedwagon in concert, and the tickets sold for $11.00 each, then estimate the total gate receipts for the concert. ________________

32. The typical American eats 96.3 pounds of beef each year. If the average price of beef is $2.42 a pound, would each spend more or less than $200 a year on beef? ________________

NSF ESTIMATION PROJECT
GRADE 8 - LESSON 13

OBJECTIVE : To apply the FRONT-END and ROUNDING strategies to problems involving mixed numbers (fractions and decimals).

TEACHER BACKGROUND

Students have already been introduced to several strategies for estimating. In this lesson we will work with mixed numbers. The first question which will be considered is:

> When is the fraction or decimal portion of a number significant enough to be considered in making an estimate?

For example, in problems (a) and (b) below, the fractional parts can be ignored without sacrificing accuracy in the estimate. Whereas in problems (c) and (d), the fractional portion in both problems is significant and cannot be ignored.

a. $38\frac{1}{4} + 78\frac{1}{5}$

c. $3\frac{4}{5} \times 28$

b. 4.037×51.7

d. $3.98 + 7.79 + 3.465$

Why can the fractional (decimal) portion be ignored in some cases but not in others?

> The bottom line is the effect it has on the estimate, which is related to the operation as well as the numbers involved. For example, in (a) a rounded estimate is uneffected by the fractional parts. However, in (d) the size of each addend is relatively small and therefore ignoring the decimal portion completely will produce more error.

This sense of appreciation for the most important part of a number is critical in estimating and needs to be continually fostered with students.

The same thinking process and strategies can be applied when estimating either fractions or decimals. Hence, in this lesson they are treated together. It is hoped that this parallel treatment of decimals and fractions will give students a better appreciation of the commonalities between them.

Two estimation strategies will be used with mixed numbers -- FRONT-END and ROUNDING. Generally, the FRONT-END processis applied to addition while the ROUNDING process is used with either addition or multiplication. In both strategies adjustments should be encouraged as estimates are refined.

TEACHING THE LESSON

PART I: In many real-life applications of mixed numbers, the fractional part of the number is insignificant. That is, it represents such a small part of the number that it can be ignored when estimating. At other times, the fractional amount is a significant part of the number and cannot be ignored.

Using TR #1, look first at problems where the fractions and decimal portions are insignificant as compared to the whole number. Students should understand that often these can be ignored. Only the whole numbers are needed to make a reasonable estimate.

$$\begin{array}{r} 156\frac{1}{7} \\ +289\frac{3}{8} \\ \hline \end{array} \qquad 49\frac{2}{3} \times 19\frac{1}{4} \qquad 94.03 + 141.968$$

Why are only the whole numbers important in making an estimate?

Help students notice that the fractions are a negligible amount as compared to the whole number. They would be difficult to work with and provide very little pay off.

Consider the next 3 examples on TR #1:

$$\begin{array}{r} 1\frac{15}{16} \\ +\ \frac{8}{9} \\ \hline \end{array} \qquad \begin{array}{r} 1.98 \\ +3.49 \\ \hline \end{array} \qquad 2\frac{9}{10} \times 1\frac{5}{6}$$

Why are the fraction and decimal parts important?

Suppose they are ignored -- how would the estimates change?

The fractional parts do make a difference here -- they are a significant part of the number and will need to be dealt with when forming an estimate. This is in part because the whole numbers are relatively small, but not solely for that reason. Consider for example the last problem on TR #1:

40.73 x 2.89

What is important?

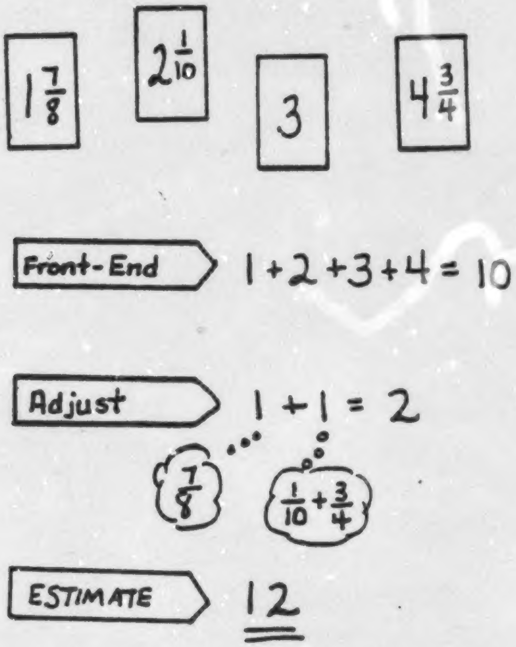
What can be ignored?

Here is an example which contains a decimal portion which can be ignored (40. 73) and one which cannot be ignored (2. 89). To estimate the answer, you might round 40.73 to 40 and 2.89 to 3 to produce 40 x 3 or 120.

Discuss each of the examples at the bottom of TR #1 with students emphasizing which parts of each problem are significant and which are not. Also mention that experience is a great teacher. As we use many different decimals and fractions our judgment of significant and insignificant parts will improve.

PART II: Ask students to use both FRONT-END and ROUNDING to estimate the sums on TR #2. As they estimate using FRONT-END with adjustment, help them group fractions easy to compute.

For example,



Use TR #2 and #3 to show students fraction and decimal estimation using both strategies. Let them discuss which strategy might be best for each problem (i.e. which is easiest, which is most accurate).

PART III: TR #4 illustrates the process of estimating products by first, ROUNDING to the nearest whole number, second, MULTIPLYING and third, ADJUSTING if possible. Equivalent decimals and fractions are used in the example (3 3/4 = 3.75 and 1 5/8 = 1.625). This illustrates further the similarity in the process for fractions and decimals.

The adjusting procedure is the same for fractions as was applied to whole numbers:

If both factors are:

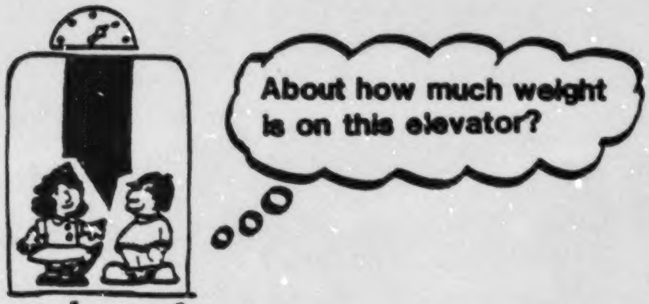
- rounded up → an over estimate results
- rounded down → an under estimate results

If one factor is rounded up and one rounded down, a closer examination is necessary to decide if the estimate is an over or under estimate. Generally, if one factor is rounded up and one down, the initial estimate is a good estimate.

ANSWER KEY

1. 8 x 9
2. 40 x 30
3. 120 + 140
4. 400 + 1600
5. 3 x 400
6. 5 x 9
7. Each fraction is less than ½ and is lost if rounding is used (because each might be rounded to 0). Since there is no "FRONT-END" whole number for these problems some grouping and adjusting is necessary.
8. If rounding is used an estimate of 9 is produced. Adjustment is needed to take advantage of the decimal portions.
9. low
10. high
11. high
12. low
13. low
14. low
15. low
16. low
17. 25-30
18. 55-60
19. 20-25
20. 11½-12
21. 20-22
22. 525-575
23. 60-61.5
24. 2-2.5
25. 25-30
26. 28-28½
27. 900-980
28. 10-15
29. 105-116
30. 19-20
31. yes
32. yes
33. $55-60
34. about $100
35. about $240

Which fraction and decimal portions are important in making a good estimate?


About how much weight is on this elevator?

$156\frac{1}{7}$ $289\frac{3}{8}$

Weights

$94.03 + 141.968$

$49\frac{2}{3} \times 19\frac{1}{4}$

3.49 lb.
1.98 lb.

About how much beef is this altogether?

$$\begin{array}{r} 1\frac{15}{16} \\ +\ \frac{8}{9} \\ \hline \end{array}$$

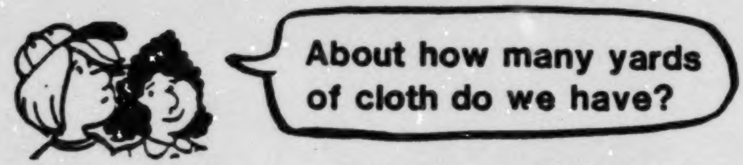
40.73×2.89

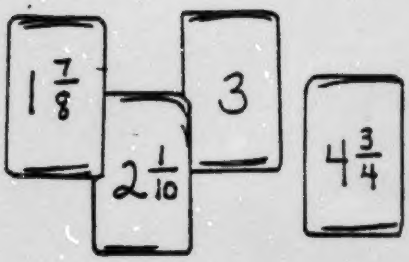
TRY THESE:

Which of the fractional parts are important?

1. $\frac{7}{8} + \frac{5}{6} + \frac{3}{4}$
2. $7\frac{1}{8} \times 3\frac{9}{10}$
3. $53\frac{1}{8} + 42\frac{1}{3}$
4. $4\frac{3}{4} \times 21\frac{1}{2}$

Adding Fractions


About how many yards of cloth do we have?


$1\frac{7}{8}$
$2\frac{1}{10}$
3
$4\frac{3}{4}$

To estimate the sum we could. . .

Round ---- $2+2+3+5=\underline{\underline{12}}$

or

Front-End --- $1+2+3+4=10$

PLUS $\frac{7}{8}+\frac{1}{10}+\frac{3}{4}$ OR ABOUT 2 MORE $-\underline{\underline{12}}$

TRY THESE:

Estimate each sum in two ways, by rounding and by front-end.

1. $4\frac{1}{3} + 2\frac{4}{5} + \frac{7}{8} + 2\frac{1}{9}$ **Rounding** ___ **Front-end** ___
2. $4\frac{4}{5} + 3\frac{1}{8} + 2\frac{1}{6} + 1\frac{1}{2}$ **Rounding** ___ **Front-end** ___
3. $2\frac{9}{10} + 3\frac{2}{5} + 1\frac{3}{8}$ **Rounding** ___ **Front-end** ___

$\frac{4}{5} + \frac{7}{8} + \frac{11}{12} + \frac{3}{4} \approx ?$

Think!
Each of the addends is close to ___ so the sum is about ___.

Adding Decimals

3.36 2.104 5.426 1.79

Estimate the total weight.

Round → $3 + 2 + 5 + 2 = 12$

Front-end → $3 + 2 + 5 + 1 = 11$

Plus about 2 more → $\underline{\underline{13}}$

Where did the 2 more come from?

TRY THESE:

1. 9.46 + 7.2 + 3.3 + 4.432 → Round:
   → Front-end:
2. 4.296 + 1.0432 + 5.42 + 0.34
3. 21.43 + 9.21 + 1.46 + 3.98
4. 1.78 + 3.04 + 0.462 + 0.89
5. 34.2 + 41.3 + 17.926

Multiplying

$\boxed{3.75 \times 1.625}$ $\boxed{3\frac{3}{4} \times 1\frac{5}{8}}$

To estimate the product. . . .

1. **Round both factors to the nearest whole number**

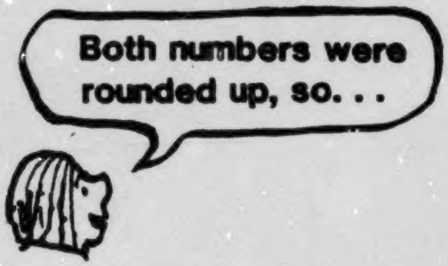
$3.75 \times 1.625 \to 4 \times 2$

$3\frac{3}{4} \times 1\frac{5}{8} \to 4 \times 2$

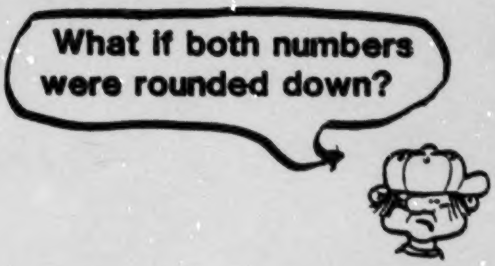
2. **Multiply**

4×2
$= 8$

3. **Adjust by thinking about how you rounded**


Both numbers were rounded up, so. . .

Less than 8

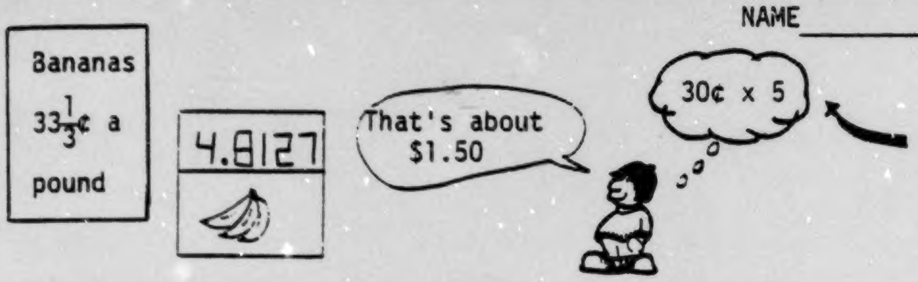

What if both numbers were rounded down?

TRY THESE:

1. $3\frac{7}{9} \times 2\frac{4}{5}$
2. $5\frac{1}{4} \times 6\frac{5}{8}$
3. $9\frac{5}{6} \times 4\frac{1}{2}$
4. 5.368×4.25
5. 27.27×8.46
6. 38.43×42.1893

What's really important?

NAME________________________



These were the numbers used.

Decide which numbers were probably used for these estimates.

	ESTIMATE	NUMBERS USED
1. $7\frac{3}{5} \times 8\frac{7}{8}$	72	________
2. 42.328 x 28.4325	1200	________
3. $119\frac{1}{9} + 138\frac{1}{8}$	260	________
4. 359.3 + 1602.71	2000	________
5. $2\frac{7}{8} \times 389\frac{7}{9}$	1200	________
6. 4.7 x 9.016	45	________

Tell why some adjustment is needed on these problems regardless of whether FRONT-END or ROUNDING is used.

7. $\frac{12}{25} + \frac{3}{7} + \frac{2}{5} + \frac{4}{9} + \frac{5}{11}$

8. 4.33 + 5.28 + .404

Look at these estimates. Then decide if the estimate is too high or too low.

9. $2\frac{3}{4} \times 5\frac{7}{8}$	10	high low	10. $\frac{7}{9} + \frac{11}{12}$	2	high low
11. $19\frac{1}{4} \times 28\frac{2}{3}$	600	high low	12. $2\frac{1}{8} \times 12\frac{3}{7}$	24	high low
13. $43\frac{1}{8}$ + 51.023	94	high low	14. $4\frac{7}{8} + 5\frac{9}{10} + 2\frac{7}{8}$	12	high low
15. 4.1075 x 3.093	12	high low	16. $5\frac{1}{9}$ x 8.023	40	high low

Estimate these.

17. $6\frac{1}{4} \times 4\frac{2}{9}$ ________

18. $9\frac{1}{7} \times 6\frac{2}{9}$ ________

19. $1\frac{4}{5} + 19\frac{3}{7}$ ________

20. $3\frac{5}{13} + 8\frac{4}{9}$ ________

21. 4.17×5.03 ________

22. 18.219×29.71 ________

23. $42.187 + 18.932$ ________

24. $.3075 + 1.825$ ________

25. $7\frac{7}{8} \times 3\frac{9}{11}$ ________

26. $5\frac{1}{8} + 23\frac{1}{9}$ ________

27. $4\frac{7}{9} \times 194\frac{7}{13}$ ________

28. $.875 \times 15\frac{3}{4}$ ________

29. $28\frac{8}{11} \times 4.06$ ________

30. $3.076 + 16\frac{2}{3}$ ________

Use estimation to solve these problems.

31. We need 10 pounds of beef for the chilli. We have 3 packages marked 3.275, 4.027, and 2.834 lbs. Do we have enough beef?

32. I bought 18.2951 gallons of gas at $1.399 per gallon. Is $30 enough to pay the attendent?

33. The coat requires $4\frac{7}{8}$ yards of material. It costs $11.87 a yard. About how much will the material cost for the coat?

34. Our wall is $8\frac{1}{4}$ feet high and $24\frac{3}{4}$ feet long. It is to be paneled with cedar which costs 49¢ a square foot. About how much will the paneling cost?

35. The hallway is $29\frac{1}{2}$ feet long and $3\frac{5}{8}$ feet wide. Carpet costs $2.04 a square foot, which includes installation. About how much will it cost for the new carpet?

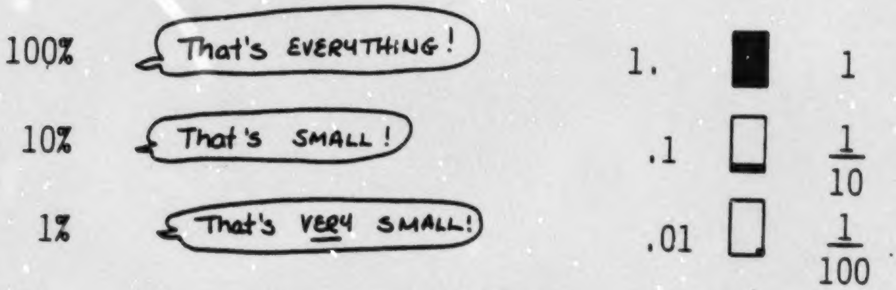
NSF ESTIMATION PROJECT
GRADE 8 - LESSON 14

OBJECTIVE : To develop estimation skills with 1%, 10%, and 100% of a number.

TEACHER BACKGROUND

Many estimation problems involve percent. Students without a clear understanding of percents become confused when computing with them. Without an intuitive feel for percents, reasonableness of answers involving percents is impossible to develop.

This lesson is devoted to increasing understanding and awareness of these three key percents.



As you can see students need a good grasp of percent, decimals, and fractions. Use of these percents in estimation situations will lead to greater understanding of percents. It will also pave the way for extending compatible numbers to percents as we have already done with other numbers.

TEACHING THE LESSON

PART I. Help students fill in the appropriate percent (1%, 10%, or 100%) in TR #1. We want them to rely on an intuitive sense for these answers. You might point out that 100% is everything, so it is the easiest to find. You might also call attention to patterns such as:

$250 \times 1 = 250$ $\qquad$ $250 \times 1 = 250$

$250 \times .1 = 25$ $\qquad$ OR $\qquad$ $250 \times \frac{1}{10} = 25$

$250 \times .01 = 2.5$ $\qquad$ $250 \times \frac{1}{100} = 2.5$

PART II. TR #2 provides a visual reminder of important mental computation patterns. Either fractions or decimals can be used. It is an individual choice, and often depends on the numbers involved.

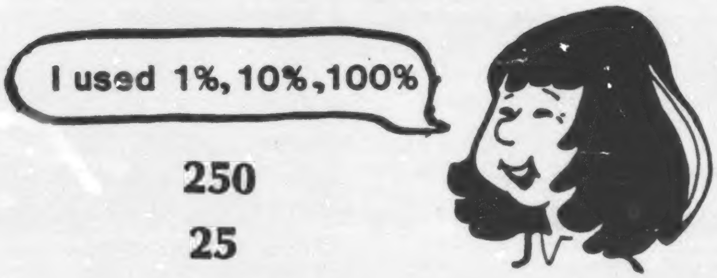
Discussion of the TRY THESE should include not only the best answer but a description of how it was determined. Hopefully students will share several different ways of deciding which answer is best.

PART III. These special percents (1%, 10%, 100%) can often be used with compatible numbers to produce good estimates. TR #3 provides some practice choosing compatibles and then making estimates. Encourage discussion of how to decide if the estimate used should be accompanied by + or -.

ANSWER KEY

1. 14
2. 45
3. 18
4. 375
5. 1.95
6. 45
7. 9
8. .75
9. less
10. more
11. less
12. more
13. more
14. less
15. less
16. $16.70
17. 8.3
18. 11,107
19. 560
20. 5570
21. 8
22. 100
23. about $240

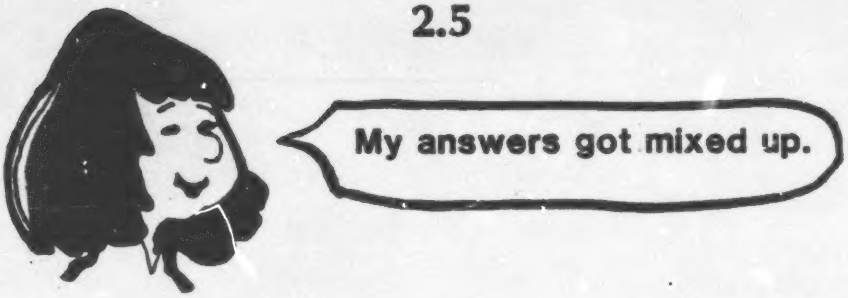
24. 15¢-18¢
25. 140-147 pounds
26. about $75

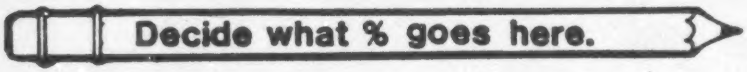

I used 1%, 10%, 100%

250

25

2.5


My answers got mixed up.


Decide what % goes here.

___ % of 250 = 250

___ % of 250 = 25

___ % of 250 = 2.5

TRY THESE:

___ of 25000 = 2500

___ of 25000 = 250

___ of 25000 = 25000

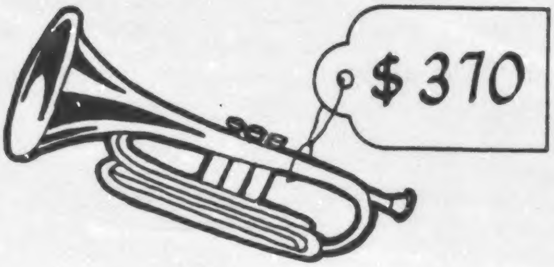
Use only
1%, 10%, and 100%.

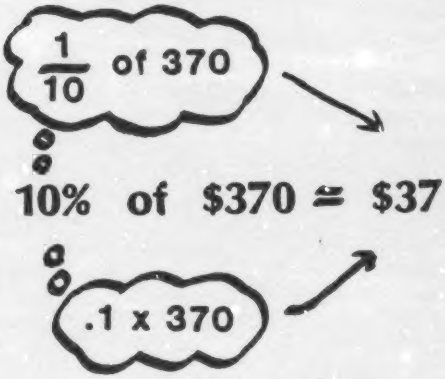
___ of 321 = 32.1

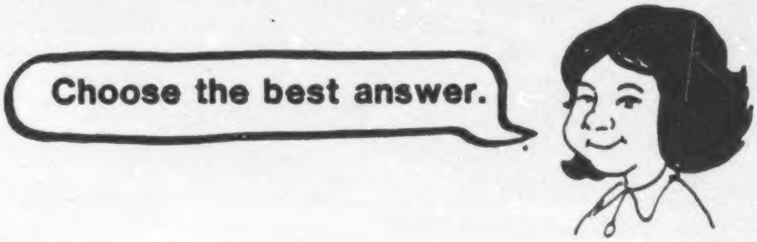
___ of 321 = 3.21

___ of 321 = 321

Save 10%



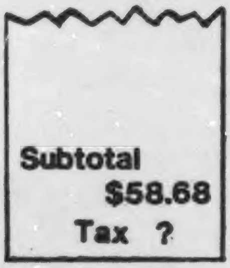

$\frac{1}{10}$ of 370
10% of $370 = $37
.1 x 370


Choose the best answer.

TRY THESE:

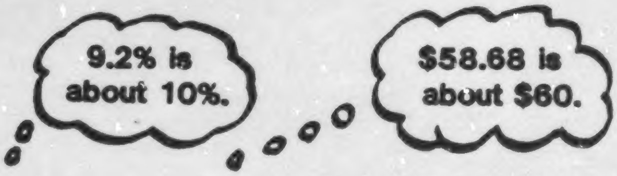
100% of 890	(8.9 89 890)
10% of $2570	($25.70 $257 $2570)
10% of $59.00	($5.90 $.59 $59)
1% of $999	($999 $99.90 $9.99)
100% of 88.80	(88.80 8.88 .888)
1% of $34200	($34200 $3420 $342)

About how much is the sales tax?


Subtotal
$58.68
Tax ?

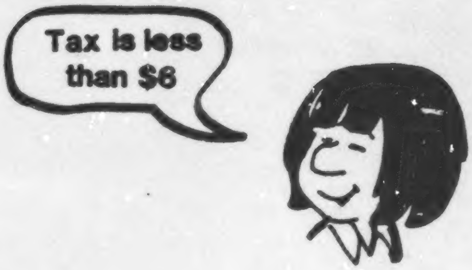
Sales Tax Rate
9.2%

1 **Choose compatibles**


9.2% is about 10%.
$58.68 is about $60.

2 **Compute** 10% of $60 = $6

3 **Adjust** $6-


Tax is less than $6

TRY THESE:

9.6% of 850 =

$1\frac{1}{16}$ % of 2800 =

98.6% of 534 =

10.23% of $670 =

$\frac{7}{8}$ % of $375 =

Name ____________________

Circle the answer.

1. 10% of 140	1.4	14	140	1400
2. 100% of 45	.45	4.5	45	4500
3. 1% of 1800	1.8	18	180	1800
4. 100% of 375	3.75	37.5	375	3750
5. 10% of $19.50	1.95	19.50	195	1950
6. 1% of $4500	45	450	4500	45000
7. 10% of 90	9	90	900	9000
8. 100% of .75	.75	7.5	75	750

A quick estimate has been found for each problem. You decide if the adjusted estimate should be more or less.

	QUICK ESTIMATE	More	Less
9. 9.65% of $45	$45	_____	_____
10. 1.03% of 855	8.55	_____	_____
11. 98.3% of 7475	7475	_____	_____
12. 10.53% of 1409	1409	_____	_____
13. 1.23% of 651	6.51	_____	_____
14. 95.3% of 11,540	11,540	_____	_____
15. 9.67% of 85	8.5	_____	_____

Which of these adjusted estimates is most realistic.

16. 9.32% of $180	$19.30	$16.70
17. 11.4% of 75	8.3	6.7
18. 97% of 11,450	11,800	11,107
19. 91 1/8% of 640	720	560
20. 9.67% of 57,600	6000	5570
21. 9/10% of 860	9	8
22. 10.45% of 960	100	90

Use estimation to solve these problems.

23. The salesman earns 9.75% of the selling price for commission. About how much will he make on a car sold for $2450. ____________________

24. The city has a sales tax of 7/8%. About how much goes to the city on a sale of $19.95? ____________________

25. Nearly 98% of our body weight is water. About how much water does a 150 pound person contain? ____________________

26. The current interest rate is 10.37%. About how much interest will a deposit of $700 earn in a year? ____________________

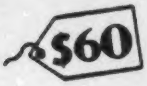
NSF ESTIMATION PROJECT
GRADE 8 - LESSON 15

OBJECTIVE : To develop skills with commonly used percents.

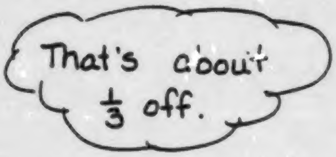
TEACHER BACKGROUND

Many real-life problems involve percents and lend themselves to computational estimation. This lesson builds onto the previous lesson but is extended to other "nice number" percents (i.e. 25%, 33 1/3%, and 50%). It involves changing percents to equivalent or nearly equivalent fractions, and using them to apply "nice number" strategies from earlier lessons. Here is an example:

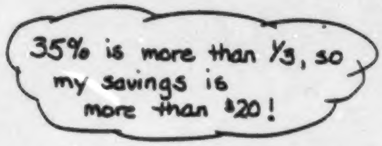
How much will you save?



35% OFF!


That's about 1/3 off.

1/3 of $60 is $20


35% is more than 1/3, so my savings is more than $20!

ESTIMATE: $\$20^+$

As you can see, once again students need to have a good grasp of percent along with general relationships between percents and fractions before doing this lesson.

TEACHING THE LESSON

PART I. Show only the vertical listing of fractions from TR #1 and ask students to copy them in the same order on their paper. Now show the entire transparency and help students match these percents with fractions. Point out that we will often find the fraction form easier to work with when estimating.

The 1/2% probably deserves special mention. Ask students for any real-world examples such as

1/2% butterfat in milk;
a special sales tax;
change in inflation rate.

Also help students distinguish 1/2% from 1% and 50%.

1/2% is less than 1%, in fact, it's half of 1%.

PART II. TR #2 shows how percents can be translated to fractional form then used with a compatible number [illegible] good estimates. Emphasize the importance of [illegible]ection of a compatible number or the fraction involv[illegible]

As this problem is being discussed, you might ask which is more important when something is being bought on sale:

> "About how much is saved from the regular price?", or
>
> "What is the approximate price paid for the item?"

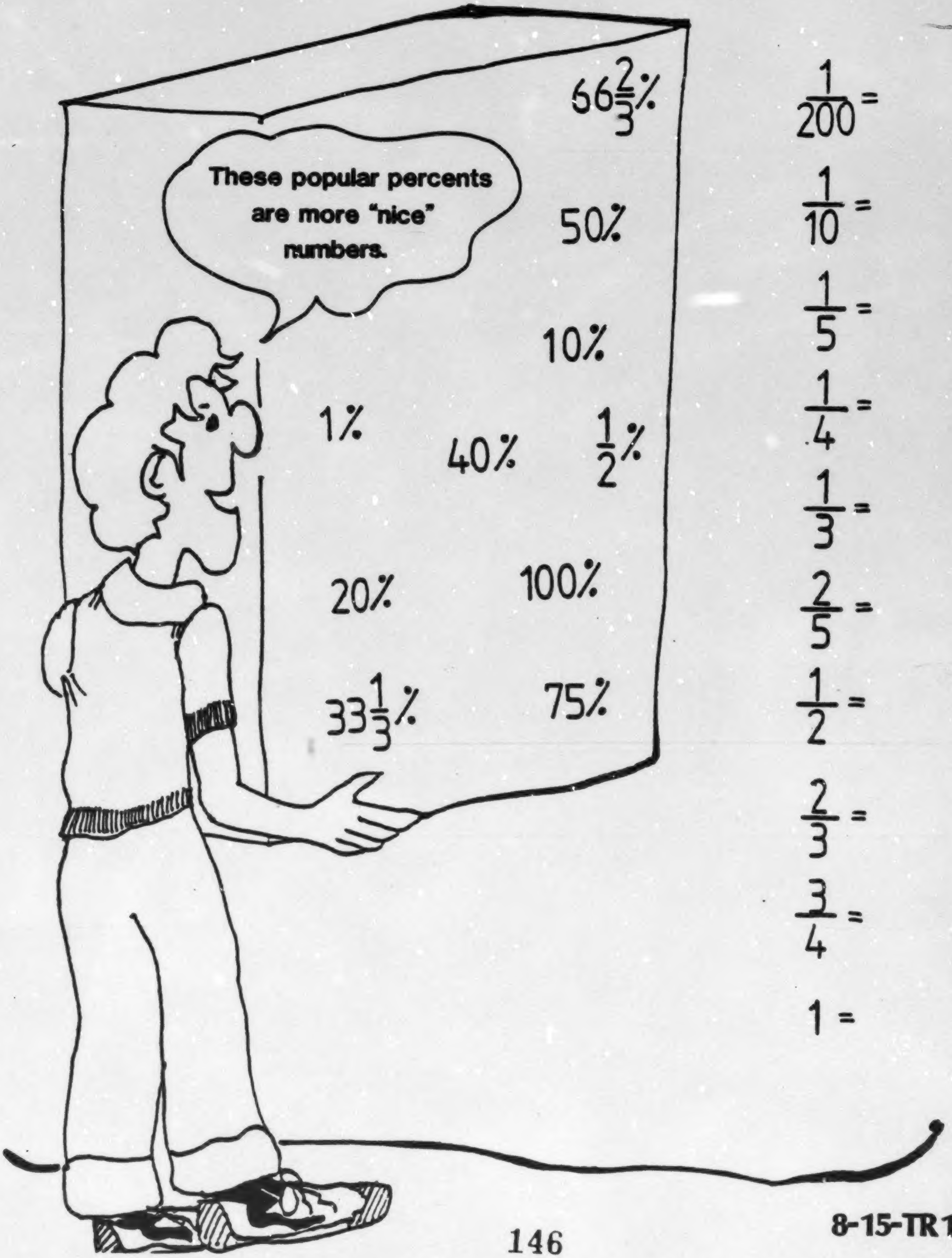
This directs students' attention to the bottom line, namely which is the better buy. Generally, good estimates are sufficient to make this decision.

PART III. Sometimes we run into messy percents, i.e. percents which do not have a common fractional equivalent such as 11½%. In these cases we will usually make use of a "nice number" percent that is close to the messy percent. TR #3 provides several applications of percents. Encourage students to search for a "nice number" percent to use that will result in a compatible set to compute with.

ANSWER KEY

1. 1/4
2. 1/5
3. 1/3
4. 1/2
5. 1/10
6. 1/10
7. 1/4
8. 1/5 or 1/4
9. 1/20
10. $80, $40
11. 1/4 of $80, $20
12. 1/10 of $80, $8
13. 1/5 of $450, $90
14. 1/10 of $900, $90
15. 1/3 of $4500, $1500
16. low
17. low
18. high

19. high
20. low
21. high
22. low
23. high
24. about 4
25. about 12
26. about 310
27. about 300,000
28. about 1,800
29. a) about $90
    b) about $300
    c) about $150
30. about $20
31. about $600
32. about $4.00
33. about $2.25,
    about $12.75


These popular percents are more "nice" numbers.
$66\frac{2}{3}\%$
50%
10%
1%
40%
$\frac{1}{2}\%$
20%
100%
$33\frac{1}{3}\%$
75%

$\frac{1}{200} =$

$\frac{1}{10} =$

$\frac{1}{5} =$

$\frac{1}{4} =$

$\frac{1}{3} =$

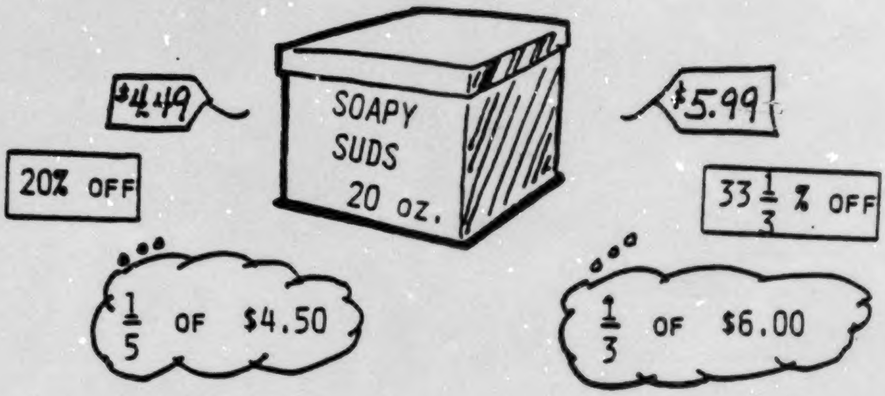
$\frac{2}{5} =$

$\frac{1}{2} =$

$\frac{2}{3} =$

$\frac{3}{4} =$

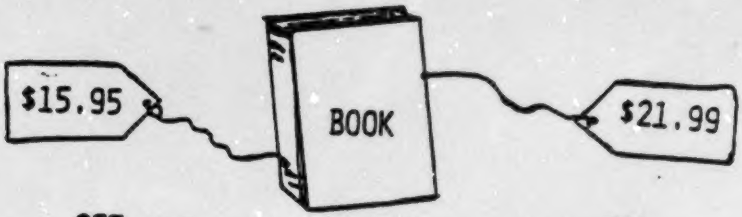
$1 =$


$4.49
20% OFF
SOAPY SUDS 20 OZ.
$5.99
33 1/3 % OFF
1/5 OF $4.50
1/3 OF $6.00

90	Save about	$2.00
$3.60	Approximate price	$4.00
✓	Better buy	

TRY THESE:

1.

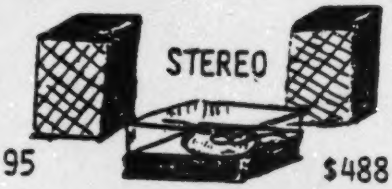

$15.95
BOOK
$21.99

25% OFF

50% OFF

	Save about	
	Approximate price	
	Better buy	

2.



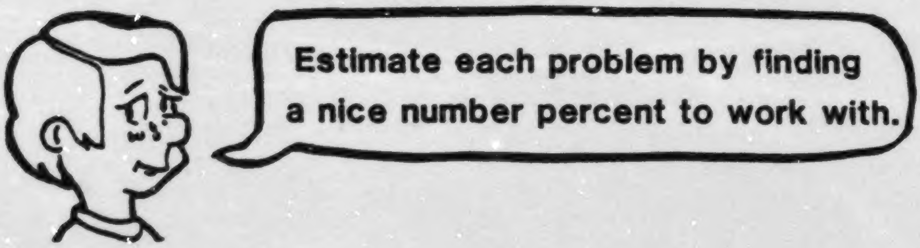
$749.95

$488.49

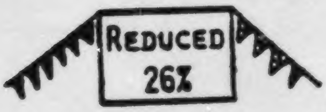
$33\frac{1}{3}$ % OFF

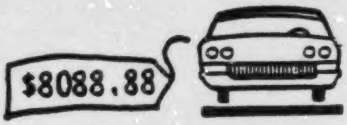
10% OFF

	Save about	
	Approximate price	
	Better buy	


Estimate each problem by finding a nice number percent to work with.

1. About how much is saved on this car?

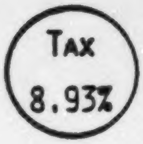


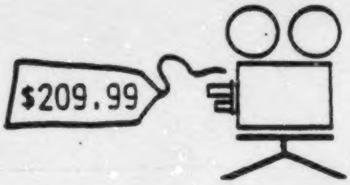


Nice numbers used: _____

Estimate: _____

2. About how much is the sales tax on this video machine?

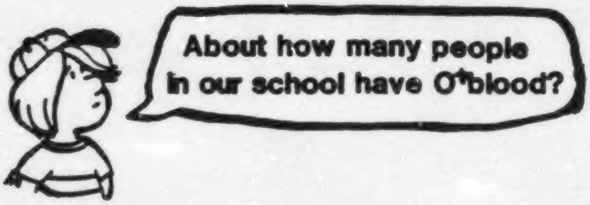




Nice numbers used: _____

Estimate: _____

3. Fact: 36% of the population has an O^+ blood type.


About how many people in our school have O^+ blood?

School Enrollment 476

Nice numbers used: _____

Estimate: _____

NAME ______________________________

Write a "nice" fraction which is approximately equal to each of these percents.

1. 24.98% ________
2. 19% ________
3. 34% ________
4. 53% ________
5. 9 1/8% ________
6. 12% ________
7. 26 3/8% ________
8. 22% ________
9. 4 1/2% ________

Chose compatible numbers you would use to make these estimates.

10. $79.99

50% off

think: 1/2 of ____

____

rough estimate of savings

11. $79.99

25% off

____ of ____

____

rough estimate of savings

12.. $79.99

10% off

____ of ____

____

rough estimate of savings

13. $444.44

19% off

____ of ____

____

rough estimate of savings

14. $888.88

10 3/8% interest

____ of ____

____

rough estimate of savings

15. $4689.95

reduced 35%

____ of ____

____

rough estimate of savings

Look at the "nice numbers" used here. Then decide if the estimate is too high or low.

1/5

16. 21% of 350 70 high low

1/2

18. 48% of 8250 4125 high low

3/10

20. 32% of 600 180 high low

1/5

22. 22 1/4% of 40,000 8000 high low

1/10

17. 10 1/8% of 1380 138 high low

1/4

19. 23% of 12,000 3000 high low

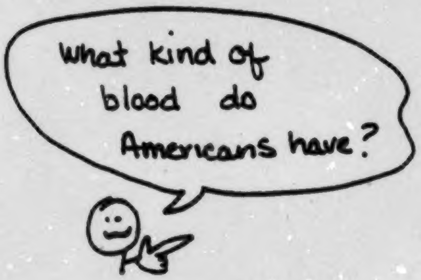
1/3

21. 32% of 600 200 high low

1/4

23. 22 1/4% of 40,000 10,000 high low

The American Red Cross reports:


What kind of blood do Americans have?

36 % have O positive blood
6 % have O negative blood
38 % have A positive blood
6 % have A negative blood
8 % have B positive blood
2 % have B negative blood
3½% have AB positive blood
½% have AB negative blood

24. On a team of 11 football players, about how many will have O positive blood? ______

25. About how many students in a class of 35 will have A positive blood? ______

26. In a city of 62,000, about how many people will have AB negative blood? ______

27. The population of Los Angeles is just over three million people. About how many people will have a blood type of either B positive or B negative? ______

Choose "nice" numbers to solve these problems.

28. The newspaper reported that unemployment had reached 9½% of the working force. Estimate the number of unemployed in a city with 18,450 eligible workers. ______

29. Waters Furniture Store is having a Clearance Sale with the following items discounted by 25%:

Recliner Chair	$ 385.00	______
Bedroom Set	1195.00	______
Sofa	595.00	______

Estimate the amount of the discounts of each item.

30. Let's suppose that you have just purchased a new stereo system for $398.00 and the sales tax in your city is 4 3/4%. Estimate the amount of sales tax on this purchase. ______

31. Thrifty Finance charges 18½% interest on loans of less than $5000 to be paid back at the end of one year. Estimate the amount of interest owed after one year on a loan of $3250. ______

32. The bill for the Ramsey family at the El Serape Mexican Restaurant was $25.82. Mr. Ramsey wants to leave the waitress a 15% tip. Estimate the amount of the tip. ______

33. Estimate the amount of discount and the new price on a tennis racket which sells for $14.95 and is advertised with a discount of 15%. ______

discount

______

sale price